On-Off Necklace Codes for Asynchronous Mutual Discovery

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Abstract—We consider mutual discovery of asynchronous wireless transceivers with a fixed activity ratio. On-off activity patterns are slotted, and repeat in discovery frames. For discovery it has to be guaranteed that the activity patterns of two transceivers are not overlapping. We design necklace codes determining activity patterns to guarantee discovery within a discovery frame, so that two asynchronous transceivers always have non-overlapping activity patterns. The number of distinct patterns is analyzed as a function of discovery frame length, and on-off activity ratio. As an application example, we consider the discovery problem for vehicle-to-vehicle communication. To guarantee discovery of far-away vehicles, discovery sequences providing processing gain, and necklace coded activity patterns are needed. We find that billions of discovery code identities can be provided with a repetition frequency that is high enough to guarantee a missed discovery probability less than $10^{-6}$.

I. INTRODUCTION

Discovery of other transceivers, possibly coupled with subsequent synchronization, is necessary in any mobile network before a communication channel can be established. In a network with a hierarchy, such as a cellular network, it is straightforward to arrange discovery by assigning the task of discovering to one layer of the hierarchy (e.g. mobile stations), and by receiving certain resources for another layer (e.g. base stations) to use for transmission of discovery signals. In ad hoc networks, all transceivers are peers, and may a priori be active in any resource. Accordingly, two non-coordinated transceivers may transmit simultaneously, leading to a collision, and failure of mutual discovery. The effects of collisions of transmissions have been investigated in the context of mutual synchronization in ad hoc networks [1], [2]. The conventional solution to mitigate collisions of beacon transmissions for discovery and synchronization in ad hoc networks is to apply randomized Medium Access Control (MAC) based on Carrier Sense Multiple Access (CSMA). CSMA solves the problem of activity pattern collisions by an inherent randomization.

The problem of CSMA-based randomization techniques is that no guarantees of discovery within a given time frame can be given, as collision avoidance is inherently stochastic. To provide reliable discovery of asynchronous transceivers with non-overlapping guarantees, is an open problem which is addressed in this paper.

The quest for reliable discovery in an ad hoc network gets contemporary motivation from V2V use cases, currently under intensive study. Vehicle-to-vehicle (V2V) communication aiming to support cooperative intelligent transportation systems (C-ITS) has been widely investigated in the context of IEEE 802.11p [3], LTE, and further in the forthcoming 5G [4]. Ongoing developments of the C-ITS applications set up increasingly rigid reliability and latency requirements. Current V2V communication is mainly for the purpose of telematics and driving assistance, whereas its future objective in 5G era is to enable autonomous (or at least highly automated) driving.

It has been envisioned that the fifth generation (5G) of wireless communication systems, can enable mission critical services in vehicular domain due to the capability to support Ultra-Reliable Low Latency Communications (URLLC) [4], [5]. Nevertheless, an IEEE 802.11p-based decentralized vehicular ad-hoc network, or a corresponding 5G development, is still an important communication technology enabling autonomous vehicles. This is due to the fact that there may be situations where the infrastructure network is down, or some specific road segments may be temporarily or persistently out of coverage of such a network.

The basic building block for the critical applications of C-ITS is a concept of cooperative awareness, where each vehicle and/or Road Side Unit (RSU) shares its own perception data obtained from its local sensors with vehicles in proximity and that allows vehicles to synchronize and coordinate their trajectories or maneuver. At a large scale, the information has to be disseminated to multiple vehicles, so that the overall traffic situation is known to all vehicles on the road. Therefore, V2V broadcasting
and geocasting are important technical enablers for future C-ITS applications.

In order to support cooperative awareness, vehicles should be able to discover each other in a fast and reliable way over a radio interface. An infrastructure based solution may be a primary one for V2V discovery and synchronization. For example, a cellular network can provide a timing reference, and aid with vehicle discovery. In an approximately synchronous Device-to-Device (D2D) setting, node discovery has been addressed in [6], [7]. In [7] signature codes that can be reliably received despite interference were considered. In [6], activity patterns were investigated, guaranteeing that a give node can discover any other node, without having to deal with full-duplex interference from itself.

To support direct V2V communication in IEEE 802.11p and possible 5G networks, especially for applications requiring high reliability, distributed solutions for fast discovery in case of asynchronous situations are needed, however. The following examples of C-ITS applications illustrate the scenarios when the fast asynchronous discovery is crucial:

- overtaking assistance, where a vehicle on a two-lane highway is to overtake another and must be informed about any upcoming traffic [8];
- intelligent non-signalized intersections, where vehicles negotiate with each other and adjust their speeds to guarantee safe and smooth passing [9];
- drive-through scenario, where the RSU provides an intermittent connectivity to the vehicles moving along the highway [10].

Here we concentrate on the first scenario, which is a challenging one for V2V communication, as the vehicles should be able to discover each other, and communicate, from distances longer than 500 m. To ensure discovery of vehicles driving in opposite directions, weak discovery signals should be decodable.

Full-duplex technology is rapidly evolving [11], but the V2V scenario considered here is particularly challenging. Directional isolation between transmission and reception antennas cannot be realized due to the scenario, the self-interference channel is rapidly changing, and the signals to be detected are very weak. Accordingly, reliable discovery is likely to be based on half-duplex methods, where transmission and reception on the same carrier cannot happen simultaneously. As all vehicles need to be able both to transmit and receive on the resources reserved for discovery, the duplexing patterns for different vehicles have to be different, to avoid possible collisions where activity patterns of two vehicles would be exactly the same, so that they would not be able to discover each other.

IEEE 802.11p adopts the random access MAC protocol CSMA type, which would not be able to guarantee discovery within a time window. In [12], packet reception probabilities of less than 20% are reported for CSMA communication with distances larger than 500 m. It is challenging to build URLLC on top of a MAC that is as unreliable. One way to increase reliability is to design the discovery patterns, where different vehicles have different patterns for transmitting and receiving discovery signals. The presented approach is applicable to improve the discovery in a wide range of mobile random access networks.

The objective of this paper is to design and analyze discovery patterns that guarantee discovery in an asynchronous setting. We address a slotted signaling model, with a fixed ratio of active slots to silent slots within a discovery frame. A somewhat similar problem has been addressed in the context of protocol sequences for the collision channel without feedback [13], [14]. In these, activity patterns are used to guarantee collision free slots in an asynchronous multiple-access channel, so that active users can be identified by the pattern used despite collisions. Here, we use the activity pattern only to discover the presence of another node, not its identity, and we are not interested in communicating data in the discovery resources.

We solve asynchronous discovery by using necklace codes that guarantee that within a discovery frame a transceiver always is silent when another transceiver transmits. These are cyclically permutable codes in the terminology of [13], but with a cyclic minimum distance $d_c = 2$, which is outside the range of the constructions in [13]. Such codes can be derived from combinatorial structures known as necklaces [15]–[17]. We discuss the number of distinct necklace codewords for different overheads, and provide performance evaluation in a long-distance ultrareliable V2V discovery scenario. We argue that within a numerology inspired by ongoing 5G developments, it is possible to design billions of distinct necklace codes with an overhead of 2% of silent slots, that enable two vehicles to discover each other with probability higher than 99.999% at a distance larger than 0.5 km.

II. System Model

The system consists of mobile transceivers that attempt to discover unknown neighbors, while simultaneously using the bulk of the resources to coordinated communication with known neighbors. The communication is a priori asynchronous, with no outside system for synchronicity. Without loss of generality, the transceivers will be called vehicles, and a V2V terminology will be used.

A. Discovery Frames

We assume that there is a discovery frame of length $N$ slots. Each vehicle dedicates $M$ Silent Slots (SS) in this frame for discovery reception only. The remaining $N - M$ slots are Active Slots (AS), where the vehicle
may communicate with peers that are known, i.e. have been discovered earlier. In addition, the vehicle broadcasts a certain synchronization signals in each active slot. These are signals with known structure, providing processing gain such that they are possible to detect with a dedicated receiver at low Signal-to-Interference-plus-Noise Ratio (SINR). For processing gain $G$, an unknown preamble has to be repeated $G$ times, or a sequence of $G$ a priori known symbols has to be used. The problem is to design asynchronous discovery patterns such that within the silent slots in one discovery frame, a vehicle will be able to discover any other vehicle using a different discovery pattern.

The basic problem is depicted in Figure 1. There are two vehicles with different discovery patterns. The silent slots are depicted as colored boxes. When the timing of colored boxes of the two vehicles overlap (even partially), both vehicles receive simultaneously, and they cannot discover each other. Such overlaps happen at different times, depending on the relative timing difference of the vehicles. Three different timings are depicted. We observe that with the depicted patterns, the vehicles will be able to discover each other irrespectively of the timing.

**B. Frame vs. Slot Asynchrony**

With slot length $T_S$, the discovery frame length is $T_F = N \times T_S$, and the timing difference $\Delta$ can, in principle, be any real number in $[0, T_F)$. First, we abstract the problem to a discrete one, by removing the effects of slot asynchrony. If the synchronization signals broadcast by the vehicles in the active slots are transmitted twice per AS, one may discretize the activity pattern design problem to one where slots are synchronous, but frames are not. If there is only one transmission, in the worst case the slot boundaries of one vehicle may always be in the middle of transmissions of synchronization signals of another. Then full synchronization signals would never be heard, and all synchronization signals would be lost. Adding another transmission removes this problem. The cost of this is increased overhead for discovery transmissions, whereas the benefit would be that in generic cases, temporal diversity is achieved for synchronization signals. An example where the two transmissions are at the beginning and the end of the slot is depicted in Figure 2.

This simplifies the design problem. The duplication of discovery broadcast transmissions in ASs renders slot asynchrony irrelevant. Timing difference may thus be abstracted to integer-valued frame-asynchronicity

$$t = \text{mod} \left( \text{round} \left( \frac{\Delta}{T_S} \right) , N \right) ,$$

which takes values $0 \leq t \leq N - 1$. This abstraction is depicted in Figure 3. It makes the discovery pattern design a combinatorial problem.

We assume that the synchronization signals themselves are specific known sequences, such as the synchronization signals used in LTE. We assume that the sequences are designed to provide a processing gain of $G$ against noise and interference. We assume that the receivers in SSs scan the channel with different timing with cross-correlators, followed by an energy detector.

Considering the slot structure, depending on the allocation of resources to the data communication and discovery purposes, there may or may not be resources for data communication in ASs. Recall that in addition to discovering unknown neighbors, the vehicles may have the need to communicate with known neighbors, and
data channels are needed for this. If Time Division Multiplexing is used between discovery and data channels, the latter would be in the ASs. In such a situation, an AS as depicted in Figure 2 may consist of two transmissions of synchronization signals only. If Frequency Division is used to multiplex discovery and data, there may be a narrow carrier solely dedicated for discovery. It is worth noting that using FDM to multiplex discovery channels in frequency does not solve the problem of activity collisions. This is because the vehicles are peers, sharing the discovery channels. On any given discovery channel, one vehicle must transmit, and another receive for discovery to happen.

III. NECKLACE CODES

In contrast to [6], we aim at asynchronous discovery. Two vehicles have to discover each other within the discovery frame of length \( N \) for any relative timing difference \( \Delta = 0, 1, \ldots, N - 1 \). The patterns thus have to be such that they differ from each other up to all possible cyclic permutations.

Cyclically permutable codes were investigated in [13]. The cyclic minimum distance \( d_c \) was defined as the smallest Hamming distance between codewords modulo cyclic permutations. Codes with \( d_c > 2 \) were designed from Reed-Solomon codes. Here, we are interested in codes with a fixed number of Silent Slots (a fixed Hamming weight), and with cyclic minimum distance \( d_c = 2 \). This is the minimum that guarantees discovery—for all cyclic permutations for codewords \( c_1 \) and \( c_2 \), there is at least one SS of \( c_1 \) not overlapping with a SS of \( c_2 \), and vice versa. For efficiency, we are also interested in codes with a small number of SSs, i.e. with small Hamming weight.

The situation for patterns with \( M = 2 \) Silent Slots can be directly analyzed. As one is interested in cyclic permutations, without loss of generality one may choose the first slot in the frame to be a SS. This is a worst case starting point—for two codewords there exists a cyclic permutation that gives the smallest cyclic minimum distance, and for this permutation there is always at least one SS collision between the codewords, which can be taken to be in the first slot. Selecting the other SS then yields a partition of \( N \) to two non-zero integers. Alternatively, the total number of \( N - 2 \) ASs is partitioned to two integer-length parts that are inserted between two consecutive SS transmissions, where a part may have no ASs. Due to cyclicity, the order of these integers does not matter: Having SSs in slots \((1, m)\) is equivalent to having SSs in slots \((1, N - m + 2)\), up to cyclic permutations. With SSs \((1, m)\), there are \( m - 2 \) ASs between the first and second SSs, and \( N - m \) between the second SS and the first in the next frame. With SSs \((1, N - m + 2)\), the number of ASs between the two SSs is exchanged. Thus, there are precisely

\[
k(N, 2) = \left\lfloor \frac{N}{2} \right\rfloor
\]

(2)
distinct discovery patterns when \( M = 2 \). These patterns are depicted in Figure 4 when \( N = 10 \).

For \( M > 2 \), the enumeration of alternatives becomes more delicate. It turns out that the unique discovery patterns up to cyclic permutations are combinatorial objects known as fixed density binary necklaces [15]–[17]. A necklace of length \( N \) is an object where \( N \) \( L \)-ary beads are threaded on a loop, so that cyclic rotations are considered to be the same necklace. In a fixed density necklace the number of each type of bead is fixed, and in a binary necklace, there are only two kinds of beads, of type, say, 0, 1. The number of binary necklaces with fixed density \( M \) is thus directly related to partitions of \( N \) to \( M \) non-zero integers. Without loss of generality, one may choose e.g. the smallest integer to be the first. After that, order matters. To construct all such necklaces, the following procedure may be followed:

1) Find all partitions of \( N \) to \( M \) non-zero integers:

\[
N = \sum_{m=1}^{M} \lambda_m, \quad \lambda \in \mathbb{Z}_+, \quad \lambda_m \leq \lambda_{m+1}
\]

(3)

2) Fix \( \lambda_1 \), and consider all non-equivalent permutations \( \Pi \) of the ordered set \( \{\lambda_2, \ldots, \lambda_M\} \)

3) Take the first SS to be in the first slot, the second in slot \( \lambda_1 \), the \( m \)-th in slot \( \lambda_1 + \sum_{k=2}^{m-1} \lambda_{\Pi(k)} \) for \( m \geq 3 \).

The number of \( N \)-element binary necklaces with \( M \) ones is [16]

\[
k(N, M) = \frac{1}{N} \sum_{j \in \mathcal{G}_{N,M}} \phi(j) \binom{(N/j)!}{(M/j)!}
\]

(4)

where \( \mathcal{G}_{N,M} \) is the set of the divisors of the greatest common divisor of \( N \) and \( M \). Here \( \phi(j) \) is Euler’s totient function, i.e. the number of positive integers \( \leq j \) that do not divide \( j \), and \( \binom{0}{\bullet} \) is the binomial coefficient.
This allows us to directly assess the number of distinct discovery patterns given \(N\) and \(M\). As depicted in Figure 5, the number of distinct patterns \(k(N, M)\) grows rapidly in \(M\). This number is maximized at \(M = N/2\). With a relatively small \(M\), the number of patterns grows to billions, when \(N \to 1000\). If the main use of the carrier is for data transmission, the density \(M/N\) is an efficiency measure—it directly describes the ratio of all resources that are available for conventional communication by a vehicle. Thus, a comparison with approximately fixed density. The number of SSs is \(M = \lfloor rN \rfloor\), so that \(M/N = r\).

Furthermore, following [15], [17], one may develop algorithmic Gray-coded enumeration of all discovery codes.

IV. V2V OVERTAKING SIMULATION

To assess the discovery performance of necklace codes, we simulate a V2V overtaking scenario.

A. Radio Communication Model

Measurement based path loss models for V2V communication can be found in [12], [18]. We use the highway Obstructed Line-of-Sight (OLOS) model from [12],

\[
PL = PL_0 + 10 n_2 \log_{10} \left( \frac{d}{d_b} \right), \tag{5}
\]

with \(PL_0 = 92.98\), path loss exponent \(n_2 = 3.18\), and break point distance \(d_b = 104\) m. Log-normal shadow fading with standard deviation \(\sigma = 6.12\) and coherence length \(d_c = 32.5\) m is assumed. Thus the shadowing coherence time for the channel of two vehicles on opposing lanes with relative velocity \(v_r\) is \(t_c = 0.59\) s. We assume fast Rayleigh fading, with mean power given by the path loss and shadow fading models.

We model interference from outside of the interference range by an Interference-over-Thermal margin of \(M_{\text{IoT}} = 10\) dB. This is a conservative estimate on par of what would be used in a cellular system [19]. A transmit power of \(P_{\text{Tx}} = 20\) dBm is used, the system bandwidth is \(W = 20\) MHz, and a receiver noise factor \(NF = 9\) dB is assumed.

B. Discovery Requirements

We consider an overtaking car driving with a velocity \(v = 100\) km/h (= 27.8 m/s). Vehicles in opposite lanes drive the same velocity, the relative velocity between two vehicles that have to discover each other is thus \(v_r = 55.6\) m/s. Assuming that the speed difference between the passing and the passed car is 10\%, and that a relative distance of \(d_p = 20\) m is covered during the passing, the passing event takes \(t_p = 7.2\) s. With a security margin of 25\%, this means that two vehicles driving in opposite directions should discover each other before the relative distance is \(d_{\text{min}} = 500\) m.

C. Simulation Results

We consider distances between \(d_{\text{max}} = 1.5\) km, which is a rough upper limit of the validity of the path loss model, and \(d_{\text{min}}\). The average SINRs in this interval range from \(\gamma_{\text{min}} = -29.6\) dB at \(d_{\text{max}}\) to \(\gamma_{\text{max}} = -15.4\) dB at \(d_{\text{min}}\).

It takes the vehicles 18 seconds to drive the interval from \(d_{\text{max}}\) to \(d_{\text{min}}\). With the discovery frame duration \(T_f = 300\) ms, which is close to a typical LTE discontinuous reception cycle duration of 320 ms [20], there are \(N_{\text{frames}} = 60\) discovery frames to discover the other vehicle. We assume that for each vehicle, there is one opportunity to discover the other in each frame, guaranteed by necklace codes.

There are \(N_{\text{shadow}} = 30\) shadow fading correlation distances in this time interval. Following [12], we model shadow fading such that within the correlation distance, shadow fades are fully correlated, and for larger distances, they are i.i.d. Thus each of the \(N_{\text{shadow}} = \)
The corresponding necklace codes have limited overhead of 2%, one can guarantee a unique synchronization signal transmission depends of the processing gain $G$ of the synchronization signals themselves, as well as the sensitivity of the cross-correlation detector searching for synchronization signals. To avoid excess signaling overhead to identify false alarms, we have tuned the threshold of the detector so that the false alarm probability is $P_f = 0.01$.

Results on detection performance are given in Table I. To achieve extreme reliability, discovery transmissions with processing gain up to $G = 512$ are needed. Furthermore, to guarantee discovery, we need necklace codes that allow each vehicle to discover every other vehicle in a discovery frame. With a slot duration of $T_S = 1$ ms in a discovery frame of $T_D = 300$ ms, the corresponding necklace codes have $N = 300$. From Figure 5 we see that one has $M = 6$ silent slots in each frame, the number of distinct necklaces is billions. Accordingly, with such necklace codes having a rather limited overhead of 2%, one can guarantee a unique necklace to each vehicle in the world.

### V. Conclusion

We have addressed the problem of designing distinct on-off patterns that guarantee that two transceivers have the possibility to discover each other within a discovery frame, so that while one transmits, the other receives. For this necklace codes of activity patterns were constructed. We addressed the usability of such discovery in an exemplary V2V scenario, where two vehicles approaching each other on a highway should be able to discover each other, and establish communication, at a distance between $d_{\text{max}} = 1.5$ km and $d_{\text{min}} = 0.5$ km. From a state-of-the-art path loss model we have found that the average SINRs would be between $\gamma_{\text{min}} = -29.6$ dB and $\gamma_{\text{max}} = -15.4$ dB. To ensure discovery in such challenging conditions, transmissions of discovery sequences providing processing gain are needed. In future work, the preliminary concept discussed here will be expanded to more refined interference and necklace code management, which would guarantee operation in scenarios with a higher traffic density.

### Table I

<table>
<thead>
<tr>
<th>$G$</th>
<th>$P_{\text{miss}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.65</td>
</tr>
<tr>
<td>8</td>
<td>0.50</td>
</tr>
<tr>
<td>16</td>
<td>0.31</td>
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<tr>
<td>128</td>
<td>0.0024</td>
</tr>
<tr>
<td>256</td>
<td>$6 \times 10^{-5}$</td>
</tr>
<tr>
<td>512</td>
<td>$3 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

30 states is realized during two consecutive discovery frames.

The probability of missed detection of a synchronization signal transmission depends of the processing gain $G$ of the synchronization signals themselves, as well as the sensitivity of the cross-correlation detector searching for synchronization signals. To avoid excess signaling overhead to identify false alarms, we have tuned the threshold of the detector so that the false alarm probability is $P_f = 0.01$.

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### References


