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Rock-Paper-Scissors

Questioning the effects of manipulative materials

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Abstract	<p>The purpose of the present study is to investigate what research show about the use of manipulative materials in the mathematics education and what factors have an impact on this approach. A definition of the problem was formed when considering the decrease of Swedish pupils mathematical abilities presented in PISA. The intention with the present study was to assemble further knowledge in whether or not the use of manipulative materials could be a teaching method that could help improve this. The method used in this study was a systematic review based on an analysis of 8 studies. The results of the studies were thoroughly examined and presented in a result chapter. When comparing the results categories were found. These categories structured the result chapter as following: <i>the beliefs of pre- and in- service teachers, the situations in which manipulative materials are used / not used, the use of manipulative materials in relation to grades and duration and pupils' beliefs and achievements</i>. Results show a positive meaning to the use of manipulatives, however it is the question of how they are used that is discussed.</p>

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Introduction

In mathematics the term “*manipulative materials*” refers to all concrete objects used as tools in mathematical education. Manipulative materials, or manipulatives, are physical objects that can be used in an explorative sense. The idea is to give the pupils something they can see, touch and examine. Manipulatives are meant to create a deeper understanding of mathematical concepts. They are often seen as a bridge that takes mathematics from abstract to concrete. An overview has been written to display what knowledge there is about using manipulatives in mathematics education: *Laborativ matematikundervisning - Vad vet vi?* (Rystedt & Trygg, 2010). This is a very comprehensive overview that assembles most, if not all, significant knowledge there is to know about the area of focus. Therefore, it came to be a big part in the first chapter (background) of this study.

In 2012 Swedish pupils’ mathematical abilities were tested in the international PISA¹ investigation by OECD². According to the Swedish National Agency of Education (SNAE³), the results have gotten worse over the last few years (Skolverket, 2013). Since the PISA investigation is quite recent, few studies have been made regarding this decreasing development. Through our own experiences we have noticed a lack of interest and motivation regarding the subject mathematics. Obviously there are many factors that can help encourage the pupils’ cognitive development. The use of manipulative materials in mathematical education could be a possible solution to these problems (Skolverket, 2013). Therefore, the main focus of this study is to investigate what research show about the use of manipulative materials in the mathematics education and what factors have an impact on this approach.

¹ PISA - Programme of International Student Assessment, an international study of 15-year old pupils’ scholastic performances in mathematics, science and reading comprehension.

² OECD - Organisation for Economic Co-operation and Development

³ Further on the Swedish National Agency of Education will be referred to as SNAE

Background

In this chapter some important aspects of the main topic are presented. The aspects consist of five subheadings; *manipulative materials*, *approaches*, *teachers' significance*, *the effects on pupils' development and perspectives on learning*. The following knowledge obtained from reports and literature has helped form *the purpose of the study* and *the definition of the problem*, which are presented in the end of this chapter.

Definition of terms used in the study

As mentioned in the introduction *manipulative materials* is a general term used to describe concrete objects in mathematics education. Rystedt and Trygg (2010) explain that these objects come in many varieties; either everyday objects (e.g. buttons and cans) or natural objects (e.g. rocks and sticks). By using these kinds of objects no time is needed for the students to get acquainted with the material as opposed to manufactured educational objects. The manufactured objects are created solely for mathematical purpose, for example Cuisenaire rods, Diene's blocks and interlocking cubes. Unlike the everyday objects, which primary function is not for mathematical use, the educational ones are designed for precisely that. Therefore, everyday objects could cause unnecessary distractions. Over the past few years manipulatives have also come to include digital objects, such as computers and tablets (ibid).

In addition to manipulative materials, *representations* is another term frequently used in this context. Rystedt and Trygg (2010) explain the term representations as forms of expression. Representations can be for example symbols, pictures or words that represent the abstracts of mathematics. Important to remember is that pupils' interpretation of representations can be different from one another (ibid). Malmer (2002) emphasizes the importance of the teacher having an open mind when it comes to which form of representations the pupils' are allowed to use in the classroom. This is of importance since it could be helpful to pupils with certain difficulties to be able to use different forms of representations (ibid). Further, Rystedt and Trygg (2010) point out the fact that there is a distinct connection between concrete and abstract in terms of mathematics, where the one does not exclude the other. In this study the term *concrete* refers to materials you can touch, see and move, while *abstract* refers to materials you can only perceive with you mind, such as tasks in a book (ibid). In order for pupils to transform their concrete comprehensions of mathematical concepts to abstract ones, Rystedt and Trygg (2010) presents a four-stage method. First, concrete work with the manipulative materials. Second, switch the concrete material to pictures, but still based on reality. Third, transform the pictures to symbols such as lines and hoops. Last, representations in form of mathematical rules and formulas. Malmer (1990) talks about a "mathematical language", which refers to pupils' conceptions of the subject mathematics. To help develop their mathematical language, it is of

importance to, for instance use concrete representations and let the pupils use all their senses to solve mathematical problems. This way they will create a firm foundation before moving on to mental calculation, which is more of an abstract approach (ibid).

Researchers tend to use different concepts to define the same phenomena. For example, Rystedt and Trygg (2010) highlights a few terms that are encountered in this area: *enactive*, *iconic* and *symbolic*. An enactive approach corresponds to the concrete, where the purpose is for the pupils to work physically and actively. An iconic approach can be compared to Rystedt and Trygg's second stage, where pictures based on reality are used. Finally, symbolic is a term that can be compared to the last stage, where formal symbols in form of rules and formulas are used (ibid).

Approaches

Rystedt and Trygg (2010) write of the fact that manipulative materials can be used in many different ways in a classroom. The status of the manipulatives is decided by both the teachers and the pupils. Depending on the attitudes towards the manipulatives the status can vary significantly. They can be used either for fun (in a non-teaching purpose) or in a more restricted way. Several studies show that some teachers use manipulatives only during short breaks in a relaxing or rewarding purpose. Meanwhile other studies show that manipulatives are being used in a more controlled way as a result of teachers fear for possible misunderstandings by the pupils (Rystedt & Trygg, 2010). In both scenarios the manipulatives lose their purpose, which as earlier mentioned is to give the pupils the opportunity to learn by exploring and creating their own experiences of mathematical learning situations.

As mentioned before, there is a four-step method that brings mathematical concepts from concrete to abstract. However, this method can be reversed. Rystedt and Trygg (2010) explain how the education in mathematics also can have a starting point in a general manner. With this approach the pupils get introduced to mathematical concepts through demonstrations of symbols, for instance in textbooks. The purpose is for the pupils to start working at an abstract level, to later apply it in a concrete context.

Another approach to manipulative material is to only make it available to the pupils with mathematical difficulties. Malmer (2002) claims that many Swedish pupils consider mathematics to be difficult. Further on, she claims that pupils who have mathematical disabilities also have difficulties seeing the abstract nature of mathematics. Therefore, it is common to provide these pupils with manipulatives. If the manipulatives are being used mainly in such purpose they can easily become associated with poor ability (ibid). Rystedt and Trygg (2010) mean that disabilities are not congenital but instead come from a limited development of inner representations. In light of all this, Malmer (1997) claims that the use of manipulative materials is a sufficient way to customize the education according to every individual's need.

Teachers' significance

Many studies show that the teacher's actions play an important part in the mathematical education (Rystedt & Trygg, 2010). The researcher Nilsson (2005) highlights that knowledge is not automatically gained just because you have worked in a laborative way. He means that it is the teacher's responsibility to clarify the goal and purpose of the activity and create opportunities for involvement and rewarding discussions, in order for the pupils to develop a deeper understanding. Rystedt and Trygg (2010) points out that the concrete tools are in fact dead material. The material itself cannot give the pupils a mathematical understanding, but needs a teacher to give it its mathematical value. Skolverket (2011) emphasizes the importance of using manipulative materials and laborative work properly. In other words, the materials and the laborations themselves should not become the main goals. Teachers should work with the materials in a way that reminds the pupils about the actual goal, which is to (with the help of the materials) understand the mathematical concepts (ibid).

Teachers' genuine attitude towards the use of manipulatives and participation with the pupils, will hopefully give a positive meaning to laborative work in the mathematical education (Nilsson, 2005). Skolverket (2003) mentions a few examples of successful educational situations, where teachers participate with an awareness of pupils' unconventional solutions to mathematical problems. In these situations teachers have prepared tasks that are based on the textbook and related to the pupils' everyday life.

Research also shows that the pupils think of the teacher as the most important factor when it comes to their will to learn. The ability to motivate, engage and mediate knowledge are all things that the pupils value (Skolverket, 2003). Malmer (1997) underline the importance of a good relationship between the teacher and the pupils. It takes communication and patience from both sides in order for the learning process to be complete.

The effects on pupils' development

In this context it is not solely the teachers that predict the level of motivation among the pupils, but it is also the pupil's own responsibility (Skolverket, 2003). From our experience pupils' cognitive development in school seems to partly depend on their passion, motivation and engagement. This is confirmed by SNAE, where similar situations have been encountered several times (Skolverket, 2003). In these situations the pupils have been given the opportunity to use both their mind and sense of touch. It is also clearly shown a joy of discovery and an all-around involvement amongst the pupils.

However, Skolverket (2011) notes that there is a distinct difference between the work method and its goal. As mentioned before, while the use of manipulatives is just the method, the goal

however is not to get the pupils activated, but for them to understand mathematical concepts. Still, when activating the pupils through the use of manipulatives motivation and engagement are fortunately often a result even if it was not initially the goal (ibid). It is affirmed that pupils' engagement increases with their involvement in the educational content and work methods. The more influence they have, the higher the level of their motivation gets. Furthermore, the purpose of mathematical education and specific tasks need to be visible to the pupils for them to be able to relate mathematical content to their everyday life (Skolverket, 2003).

Definition of the problem

The alarming PISA results have been shocking to the Swedish society. Our knowledge in mathematics has always been considered important. When a comparison of the Swedish PISA results was made between 2003 and 2012 a continuous decrease was detected. Since no change for the better have been made it is obvious that something has to be done. All kinds of methods are therefore now researched and tested with the purpose to improve the pupils' mathematical knowledge. Practical work and the use of *manipulative materials* are examples of such methods.

Purpose of the study

The purpose of this study is to investigate what research show about the use of manipulative materials in the mathematics education and what factors have an impact on this approach. With this study we want to find out what makes a difference in mathematical education (both teaching and learning) when manipulatives are being used and not used.

- What does research show about the use of manipulative material in the mathematics education?
- What factors have an impact on the mathematics education when using manipulative material?

Method

This chapter presents the different approaches and choices of methodology in the systematic review. Eriksson Barajas, Forsberg & Wengström (2013) explains a systematic review as a study to immerse oneself in a certain area, by examining and identifying significant themes in relevant research to one's purpose. The methods of the study can be divided into two parts. First, we gradually narrowed the search by using more specific keywords in order to induce studies that were relevant to the purpose of this study. Second, we determined whether to choose or discard studies by reading their abstracts and then examined which studies were most relevant to our purpose. To help present the methodological choices this chapter is organized into the subheadings *search for studies* and *coding of studies*. A table of search history is also shown to

further clarify the procedure. Significant to mention, the searches were made separately whilst all the examining and reading of the studies were done together.

Search for studies

Initially, a general search was made in various databases in order to obtain a wide set of concepts relevant to the chosen topic. Further a systematic search was made in the educational oriented databases ERIC⁴ and JSTOR⁵. In order to specify the search, truncation of the keywords was added. In addition, the search in ERIC was limited to only peer-reviewed results while JSTOR was limited to only show articles and reviews. References from popular publications and reports were also used in the search for relevant studies to include in the review. The final searches provided 172 adequate results. In the first sampling the titles of all the results were examined. During the scanning it was taken to account that the titles do not always give a fair impression of their content. For example some titles were identified as quite distant to our subject but still had components that aligned with our concepts. With an open mindness like this, 47 articles were eventually identified as potentially relevant according to the purpose of the present study (see sample 1 in Chart 2).

Further, all the abstracts were scanned for relevance. Of these, 24 articles were either unavailable in full text or did not cohere with the purpose of the study. For example, articles concerning only one student's development or a certain disability were excluded due to their limited area of focus. The remaining 23 articles were thoroughly screened for eligible studies to be selected for the second sample. Lists for screening processes from Eriksson Barajas, Forsberg & Wengström (2013) were used as guidelines to determine whether or not the articles were scientific. The authors explain the structure of a scientific study as follows; abstract, introduction, purpose, method, result/analysis, discussion and references. Of the remaining 23, 7 articles met the inclusion criteria and were selected to be included in the review. In addition to these 7 articles one more was found in a reference list of a report and was chosen for its relevance to the present study (sample 3, chart 2).

Coding of studies

According to Eriksson Barajas, Forsberg & Wengström (2013) it is important to include all results from the chosen articles that coheres with the purpose of the study and the researchers are not supposed to involve their own opinions in the presentation. Therefore, all results presented in the analysis of this study are strictly free from personal values. The articles were re-read separately in order to create individually perceptions. Further, the results were discussed and

⁴ ERIC - the Education Resources Information Center

⁵ JSTOR - (short for Journal Storage) a database that focuses in the areas of social and natural science and humanities.

analyzed between one another to exchange interpretations, this to obtain a more extensive understanding. Only articles relevant to the purpose of this study and met the criteria for inclusion were included in the review. The articles included are presented in an overview which illustrates their purpose, method, sample/loss and result/conclusion (appendix B).

Initially, the articles were analyzed independently and their contents categorized by both their similarities and differences. The content of the articles that was considered valuable to the present study was coded. Later on, the coding developed into themes which became the foundation of the presented result. The themes of the result chapter consist of: *the beliefs of pre- and in-service teachers, the situations in which manipulative materials are being used, the use of manipulative materials in relation to grades and duration, pupils' beliefs and achievements.*

When doing the systematic review the main approach was to evenly share the work. The only part that was done separately was the search of studies. All interesting findings were noted and saved in a shared document for further examinations. All the reading and discussing of articles were done together. Additionally, the structuring of the study was decided together as well.

Inclusion criteria

In order to be included in the review the articles had to meet the following criteria.

1. include practical operations in mathematical education
2. treat either teachers' or pupils' beliefs about the use of manipulative materials
3. consist of at least 10 participants
4. peer-reviewed

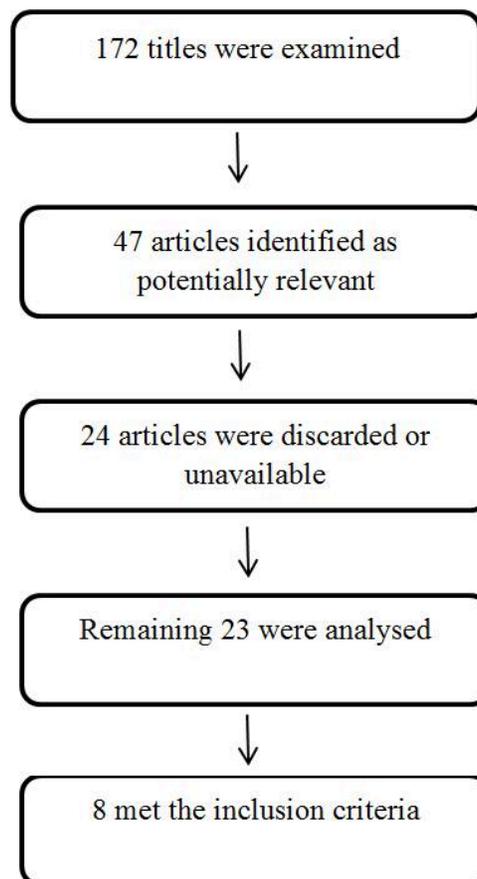


Chart 1. Flow-chart of screening process

Results

The studies analyzed in this chapter were coded for the features: different factors that determine if manipulatives are being used or not, and the resulting effects. Through this process, categories were formed with the purpose of the present review as a motive. These categories have structured this review and are presented in this chapter as four headlines. First, the beliefs of pre- and in-service teachers, where it is emphasized how the teachers' beliefs characterize the mathematics education, including the use of manipulative materials. Second, the situations in which manipulative materials are being used and how they affect the pupils' attitudes toward it. Third, the use of manipulative materials in relation to grades and duration. Fourth, the results related to the pupils' beliefs and achievements are presented.

The beliefs of pre- and in-service teachers

How much manipulatives are being used in mathematics education depends a lot on what beliefs the teachers hold (Moyer, 2001). Research on this discusses different reasons for teachers not wanting to use manipulatives, for example: the fear to create chaos (Moyer, 2001) and the belief that the necessity of using manipulatives decreases the older the pupils get (Uribe-Flórez & Wilkins, 2010). Also, Barnett and Eastman (1978) emphasize the importance of pre-service teachers' beliefs and intentions since they are the ones who will educate the pupils in the future.

Results from the study by Moyer (2001) show that some teachers think of manipulatives as tools used only when having "fun maths", while "real maths" requires more structured lessons. For example, during an interview one teacher pointed out that in order to make the pupils understand the concept she felt like she had to use pencil and paper, otherwise they would not learn it "right" (ibid). In contrast to this, 314 (90%) pre-service primary teachers in Australia claimed they intended to provide concrete material for the pupils to touch in either 75 % of lessons or every lesson (Scott, 2005).

Moyer (2001) had the intention of explaining how and why the teachers use manipulative materials in the mathematical education. In the study she explains that since teachers' beliefs reflect on their approaches, the pupils' attitudes toward manipulatives are based on these beliefs (ibid). For example, in a study by Levenson (2010) many pupils claimed that they preferred mathematical explanations over practical, due to the concision of the explanations. Some of them thought of mathematical explanations as easy, short and clear. Contrary to this, further results from the study by Levenson (2010) show that other pupils thought practical explanations were more fun and one pupil even stated that, when using practical explanations they did not really need to do any calculations. Uribe-Flórez & Wilkins (2010) state a rather evident conclusion: teachers with the notion that it is important for the pupils to work in a concrete and explorative way consequently use manipulatives more frequent. With this taken to account Moyer (2001)

explains that by using manipulatives only for “fun” the pupils don’t get the chance to explore “real maths” in appealing ways.

Barnett and Eastman (1978) performed an experimental research where 78 pre-service teachers contributed. The goal was to test the hypothesis that mathematical ability and achievement does not depend on whether or not the operations are completed in both an enactive and iconic fashion, or only in an iconic. Using a pre- and posttest it was proved that there was no significant difference between the two approaches. After the mathematics course, the results of the posttest showed no significant difference in the group that had only worked iconic as the one who had also worked enactive. A result of this experiment worth mentioning is that several of the subjects that only operated in an iconic mode indicated that they missed concrete material to work with (ibid).

Barnett and Eastman (1978) mean that pre-service teachers should get to operate in the same fashion that we expect them to teach later on. The study by Scott (2005) shows that over 90 % (324) of the pre-service teachers who participated intended to use concrete material when introducing a new concept, in either 75 % of lessons or every lesson (Scott, 2005). During interviews it seemed like the pre-service teachers remembered having used manipulatives in their own schooling, although they did not seem to remember the mathematical content from these situations (ibid).

The situations in which manipulative materials are used / not used

As mentioned before, to some teachers using manipulatives is associated with the purpose of having fun, not learning. Therefore, in interviews teachers stated that they used manipulatives as a reward when the pupils had behaved properly in class (Moyer, 2001). There were also situations where teachers saw the use of manipulatives as a privilege. When the pupils misbehaved the teachers took away the manipulatives as a punishment. In a similar manner some teachers restricted the use of manipulatives to only breaks or at the end of the lessons (ibid).

Several studies show that pupils’ preferences of using manipulatives vary in different mathematical situations and contexts. In the study by Levenson (2010) it is shown that pupils preferred practical explanations when it comes to parity. On the other hand, in a questionnaire concerning fractions, no significant preferences were stated for either practical or mathematical explanations. Meanwhile, a study by Thompson (1992) shows examples of pupils who were not quite comfortable with practical methods when it comes to solving addition and subtraction problems. In a meta-analysis by Sowell (1989) it is suggested that pupils’ attitudes toward manipulatives depend on the design of assignment. Through an investigation it was discovered that pupils’ preference of concrete versus abstract instruction depended on whether or not the

groups are assembled randomly. In other words, if the assignment were not randomly made, the pupils' attitudes were negative and they preferred abstract instructions.

The use of manipulative materials in relation to grades and duration

Several studies show that both level of grade and duration have significant roles in the use of manipulative materials. In the study by Uribe-Flórez & Wilkins (2010) 503 in-service teachers were subjects of a survey with the purpose of investigating if the use of manipulatives differs in level of grade, from kindergarten to the fifth grade. Results show a significant mean difference, in regards to the use of manipulatives in the level of grades. The use of manipulatives appears to continuously decrease in higher grades. Kindergarten teachers used manipulatives most frequently, while grades 3-5 teachers used them the least. Further, the teachers were asked if they consider older pupils to have the ability to reason abstractly and therefore do not need to use manipulatives. The results of the question cohere with the mean difference in the level of grades. The teachers that agreed to this consequently did not use manipulatives as much as the teachers that did not agree (ibid). Regardless, in a study by Norton and Windsor (2008) the majority of the seventh graders participating thought of the manipulatives as meaningful. The study investigated whether or not the use of manipulatives helped pupils understand algebra and also if their confidence in high school algebra was improved by using them. Results show that the pupils confidence enhanced as a result of their increased competency (ibid).

Further, research show that positive effect on pupils' achievements is related to the duration and frequency of the use of manipulatives. Sowell (1989) presents how the use of manipulatives in instructions had the most profitable effect when used for a year or longer on a regular basis. In Thompson (1992) observations were done regarding pupils attitudes and understandings concerning the use of different mathematical methods. However, no differences could be discussed due to the short length of the study (9 days).

Pupils' beliefs and achievements

The teachers' beliefs control their way of teaching and this can reflect on the pupils' beliefs and achievements. Several studies emphasize some complications that could occur while using manipulatives. As in every teaching situation, pupils learn in different ways and the use of manipulatives is not an exception. Moyer (2001) highlights that when introducing a mathematical concept through manipulatives, pupils will interpret and use the representations in many different ways. These interpretations may or may not coincide with the teacher's intentions. Through interviews pupils in the seventh grade stated that manipulative are helpful when a new concept is introduced. They said things like: see the structure, get a picture in your mind. However, once they understood the concept they preferred working abstract from there on.

Results of classroom observations show that the pupils working with the manipulatives had trouble writing down on paper the abstraction that the manipulatives represented (Thompson, 1992). On the other hand, in the study Levenson (2010) one pupil explained that he or she preferred abstract explanations to mathematical problems, because it was easier to explain something using mathematical sentences. Sowell (1989) examined pupils' preferences of pictorial instruction versus abstract explanation. However, in contrast to Levenson's (2010) results where several pupils praised the pictures connected to the assignments, Sowell's result showed no significant effects of the pictorial instruction.

Discussion

In this following chapter the method and the results of the study is discussed. By connecting the background chapter with the results we critically reflect and discuss its content. The methodological choices of the study is also critically discussed and assessed.

Discussion of method

When doing any kind of study Bryman (2011) talks about the importance of a couple of criteria you need to take into account as a researcher. These criteria are validity and reliability. Therefore we considered these when doing all of our methodological choices. Bryman (2011) explains external validity as a measure of the study's ability to be generalized. A study with high external validity is one where the results could be applied elsewhere. We found many studies that had a small number of participants. Since these studies' generalizability could be doubted they were excluded. Hence, one of our inclusion criteria became no less than ten participants. On the one hand we believe that this study have a high degree of validity due to the fact that the choices and results answer to the original research questions. On the other hand the large geographical distribution and the small number of participants in each study may decrease the degree of external validity.

Through our own experiences and education we have noticed a growing interest for the use of manipulatives over the last years. Even so, we found that many studies have already been made in this area over the last decades. With this discovery, we decided no limited interval when it comes to the year of publication was necessary. There may be both benefits and disadvantages when it comes to using older research for a present study. However, we decided that it was more interesting to investigate what research show about manipulatives over a longer time than over the most recent years. From the included studies 1 were quantitative, 4 were qualitative and 3 were a combination of both. As Bryman (2011) explains quantitative studies provide results that could be put in a bigger context due to their large number of participants. Meanwhile the qualitative studies can give a deeper understanding of, in this case, pupils' and pre-/in-service teachers' beliefs (ibid). The fact that the methods of research varied, may have raised the reliability of this study. To further raise the reliability, all the studies were screened for biased tendencies in the pursuit of retrieving only value-free results. To make sure the studies we chose were of high scientific quality they were screened using checklists of Eriksson Barajas, Forsberg and Wengström (2013). All studies chosen for the final sample showed the characteristics of a

scientific study. As mentioned before, according to the authors a scientific study contains these sections: abstract, introduction, purpose, method, result/analysis, discussion and references (Eriksson Barajas, Forsberg & Wengström, 2013).

Discussion of results

Many different factors matter in the mathematical education and the use of manipulative materials. The teachers' role is both significant and essential to the pupils' development (Rystedt & Trygg, 2010). In similarity, teachers' influence and engagement is according to Moyer (2001) one of the things that mirrors the pupils' achievements and development. Since the teachers have such big influence it is of importance that they have a meaningful purpose with every choice and action (ibid). With this in mind, it is the purpose of the different approaches that is interesting when it comes to the use or non-use of manipulatives. Both Skolverket (2011) and Rystedt and Trygg (2010) explains the purpose of manipulatives is to learn abstract mathematical concepts in an actively and exploring way. However, several studies show that not all teachers obtain this philosophy (Moyer, 2001; Uribe- Flórez & Wilkins, 2010). Some teachers think the only way to learn a new concept is in a traditional way with the use of pencil and paper (Moyer, 2001). Meanwhile other teachers do not take the actual mathematical content into account and instead use manipulatives with the purpose of letting the pupils play and let of steam (ibid). Further Skolverket (2011) emphasizes the importance of not putting laborative work on a pedestal and think the learning process will come automatically. In unison, Rystedt and Trygg (2010) explains that it is the teachers role to give the material its mathematical value. Both Nilsson (2005) and Moyer (2001) consider that the teacher's attitude against laborative work affects the pupils' opinion about it.

Over 90 % of the pre-service teachers participating in the study by Scott (2005) were certain that they were going to use manipulatives in either 75 % of lessons or every lesson. Questions to consider are on what basis and with what purpose? Skolverket (2011) also highlights the importance of setting essential goals for the work with manipulatives. Thus, the manipulatives and the activities connected to them should not become the main focus of the lessons. This is confirmed by Thompson (1992) whose analysis indicates the fact that in order to make valuable use of concrete materials, the pupils first need to learn how to use them in relevant manners. Further, for the use of materials to be meaningful for the pupils the mathematical knowledge needs to be firmly established. A rather concerning issue is the results shown in the study by Scott (2005). That is, most of the pre-service teachers remember having used manipulatives in their own schooling, but evidently could not remember the mathematical concepts connected to the material (ibid). This gives us a reason to question the use of manipulatives and whether or not it is meaningful. The manipulatives lose their meaning if the mathematical content is not established.

In the study by Thompson (1992) it is shown that pupils saw the work with the manipulatives as something separate from what they wrote on paper. With the four-stage method explained by Rystedt and Trygg (2010) in mind, the process reported in the study by Thompson (1992) could be seen as insufficient. This because the pupils have to put their concrete work onto paper in an abstract context directly. According to the method of Rystedt and Trygg (2010) it is profitable to go through a few steps in between. First, with the help of pictures and then also symbols to establish the knowledge before transforming it to an abstract form (ibid). The problem described in the study by Thompson (1992) could be one of the possible grounds for why the teachers participating in the study by Moyer (2001) considered the pupils to only be able to learn in the “right” way if they were constrained to the use of pen and paper. However, in the study by Levenson (2010) several pupils stated that they preferred to get instructions in an abstract form. One pupil explained that it is easier to understand the concept when explained with a mathematical language (ibid). According to Rystedt and Trygg (2010) this reverse method could also be favoring. With this method the mathematical content can be taken from a specific context to then get applied in a greater picture.

The psychologist Vygotskij believed that people learn from each other, for instance through imitations (Rystedt & Trygg, 2010). Taking this into account, it may be one of the reasons why pupils in the study by Sowell (1989) preferred concrete instructions in randomly assembled groups. Apparently, in non-random groups abstract instructions were significantly more preferred. A plausible explanation to this result may be that non- random groups provide a safer and more open environment. In these groups the pupils might feel more confident to work in an abstract way. In similarity, Norton and Windsor (2008) confirm that social and collaborative work has a positive effect on pupils’ confidence in their abilities. Research show that the use of manipulatives generally decreases the older the pupils get. The teachers seem to think that older pupils possess a more abstract way of thinking and therefore do not need to use manipulatives (Uribe-Flórez & Wilkins, 2010). Likewise, the seventh grade pupils participating in the study by Norton and Windsor (2008) found the use of manipulatives meaningful when first getting familiar with a mathematical concept. The fact that pupils think it is easier and faster to work in an abstract way, could be a sign that they have further developed an abstract comprehension (Rystedt & Trygg, 2010).

Malmer (2002) mentions that manipulatives can easily become naturally associated to pupils with low abilities in mathematics. Rystedt and Trygg (2010) highlight that these pupils, however, were not born with difficulties, but have not yet fully developed their inner representations. To strengthen this assumption, it is significant to mention Levenson’s (2010) study, which proves that medium- and low-ability pupils are fully capable of both using mathematical explanations and even generate them on their own. However, the study also shows that when it comes to which explanation the pupils find most convincing, high-ability pupils prefers abstract while the low- and medium-ability pupils chose concrete explanations (ibid).

When we analyzed the differences in all the studies depending on their year of publication few findings were made. When it comes to the factors that influence the use of manipulatives in mathematics education no significant differences were found over the thirty years. The only visible difference among the studies over time was the development of new material. Hence, the use/ non-use of materials does not seem to depend on which forms the materials have, but the different aspects of teaching methods.

The purpose of this study was to investigate what research show about the use of manipulative materials in the mathematics education. If the majority of research proves that manipulatives always is the better method, perhaps it could be used to improve the PISA-results in the future. According to the studies included in this analysis however, that is not the case. A couple of studies show no significant differences in achievements and preferences when testing both concrete and abstract work methods, among pupils and pre-service teachers (Barnett & Eastman, 1978 & Levenson, 2010). Studies also show positive effects of using manipulatives both in the classrooms with pupils and in pre-service teachers' education (Levenson, 2010; Barnett & Eastman, 1978; Thompson, 1992; Norton & Windsor, 2008 & Moyer, 2001). Although, in order to get positive results, these studies suggest that manipulatives should be used in the right manner and the goals of using them should be clear. With this said, one possible reason to why there are problems in Swedish mathematical education may not relate to the question of using or not using manipulatives. Instead it should be asked in which ways manipulatives are being used.

Conclusion and implication

Through the results of this study it is indicated that manipulatives can be used with a positive effect in all situations. However, situations differ due to the influences of many different factors. For example, the extent to which the manipulatives are needed as a help depends on where the pupils are in their development. If the pupils have not fully developed their inner representations the use of manipulatives could be of importance. In contrast, for the pupils who have come further in their development, manipulatives might be less necessary. Furthermore, both the purpose and the goal of the use of manipulatives should be thoroughly considered, for it to be a profitable method. Research also suggests that regularity of use in education has a positive effect on pupils' learning. Finally, it is the beliefs and attitudes of both teachers and pupils that decide the climate in the classroom. With this said, research does not favor only concrete work. After all, results show no differences in mathematical abilities regarding the concrete and abstract approaches. There should be a proper balance in the mathematical education, where concrete and abstract are equivalent.

The results of this study have induced an awareness of the lack of research in some aspect of this area. For example how to proceed in order to successfully plan and execute lessons integrating

manipulatives. Also, as the technology develops, further research where the effects of everyday and natural manipulatives are compared to digital manipulatives is welcomed. With this study we hope to contribute a meaningful insight of which factors to consider regarding manipulative materials.

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*Result articles

Appendix A

Chart 2. Search History

Date	Database	Keyword	Hits	Sample 1	Sample 2	Sample 3
2014- 11- 11	Google Scholar	laborative mathematics	9 950			
2014- 11- 11	Google Scholar	concrete mathematics	968 000			
2014-11-17	Google Scholar	manipulative materials	99 800			
2014-11-17	Google Scholar	manipulative materials mathematics middle school	34 000			
2014-11-17	Google Scholar	manipulatives	30 000			
2014-11-17	ERIC	manipulatives	3 093			
2014-11-18	ERIC	math* manipulative material concrete	149			
2014-11-18	ERIC	math* manipulative material concrete Limits: peer reviewed	32	24	7	2
2014-11-24	JSTOR	"manipulative material" mathematic*	27	9	6	2
2014-12-04	JSTOR	"concrete materials" manipulative* mathematic* Limits: Articles & Reviews	113	14	10	3

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
2001	ERIC	Patricia S. Moyer	Are we having fun yet? How teachers use manipulatives to teach mathematics	The purpose of this study was to explain how and why teachers use manipulatives, for mathematics instruction, as they do.	A qualitative study that consists of observations, interviews and self-report data	Sample: A purposive sampling of 10 in-service teachers. Loss: 8 in-service teachers	Results show that manipulatives are being used in different ways and with different purposes by the teachers. Some teachers only use manipulatives for “fun”, during breaks or at the end of the class.
Educational Studies in Mathematics	"concrete materials" manipulative* mathematic* Limits: Articles & Reviews						

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
1989	JSTOR	Evelyn J. Sowell	Effects of manipulative materials in a mathematics instruction	The purpose of this study was to integrate the findings of research on the effects on student achievement and attitudes of using manipulative materials in mathematics instruction.	A meta-analysis of studies comparing the results of equivalent comparison groups, either a concrete (or a pictorial) treatment with an abstract treatment or a concrete treatment with a pictorial treatment.	Sample: 60 studies met the inclusion criteria. Loss: 58 studies	This meta-analysis could not answer questions about which educational situations manipulatives might be appropriate.
National Council of Teachers of Mathematics	"manipulative material" mathematic* No limits						

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
2010 School Science and Mathematics	ERIC math* manipulative materials attitude Limits: peer-reviewed	Lida J. Uribe-Flórez & Jesse L. M. Wilkins	Elementary School Teachers' Manipulative Use	The purpose of this study is to investigate how teachers grade level, beliefs about manipulatives, and other background characteristics affect how often elementary school teachers use manipulatives during their mathematics instruction.	A quantitative study that consists of a survey.	Sample: 503 teachers completed all the items from the questionnaire. Loss: 27 teachers did not complete them all.	The results show that the teachers' beliefs and grade level were significant to how often manipulatives were used in mathematics instruction. However, the teachers' background seems to have no significant impact on the use of manipulatives.

Appendix B:4

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
1978 National Council of Teachers of Mathematics	JSTOR "manipulative material" mathematic* No limits	Jeffrey C. Barnett & Philip M. Eastman	The use of manipulative materials and student performance in the enactive and iconic modes	<p>The purpose of this study was to test the hypothesis;</p> <p>in measures of ability to demonstrate numerical and structural properties of the four basic arithmetic operations in the enactive mode / iconic mode, there are no significant differences between subjects who are required to operate in both the enactive and iconic modes and subjects who are restricted to operating only in the iconic mode.</p> <p>In measures of mathematical achievement, there are no significant differences between subjects who are required to operate in both the enactive and iconic modes and subjects who are restricted to operating only in the iconic mode.</p>	A qualitative study that consists of an investigation.	<p>Sample : 78 elementary education majors.</p> <p>Loss: 8 elementary education majors</p>	The results show that the experimental subjects (enactive mode) did not show any superiority over the control subjects (iconic mode). No significant differences were found between the groups on the laboratory portion of the posttest and interviews, but the control group completed the laboratory exercises in approximately 25% less time than did the experimental group.

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
1992 National Council of Teachers of Mathematics	JSTOR "concrete materials" manipulative* mathematic* No limits	Patrick W. Thompson	Notations, Conventions, and Constraints: Contributions to Effective Uses of Concrete	The purpose of this study is to investigate the features of students' engagement in tasks involving base-ten blocks contribute to students' construction of meaning for decimal numeration and their construction of notational methods for determining the results of operations involving decimal numbers.	Qualitative study that consist of an investigation	Sample: 20 fourth-grade students No loss	There are two lessons suggested by this analysis and the results of the study. The first is that before students can make productive use of concrete materials, they must first be committed to making sense of their activities and be committed to expressing their sense in meaningful ways. The second is that for concrete embodiments of a mathematical concept to be used effectively in relation to learning some notational method, students must come to see each as a reflection of the other-constraints and all. They must end up feeling just as constrained in their notational actions as they do with those actions' counterparts in a concrete setting.

Appendix B:6

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
2010	JSTOR	Esther Levenson	Fifth-Grade Students' Use and Preferences for Mathematically and Practically Based Explanations	The purpose of the study was to investigate fifth-grade students' use and preferences for mathematical based and practically based explanations within two additional contexts: parity (the property of being even or odd) and equivalent fractions.	Both a quantitative and qualitative study that consists of questionnaires and student interviews.	Sample: 105 fifth-grade students. No loss	Results showed that significantly more students were most convinced by PB explanations and preferred to use a PB explanation when explaining a mathematical concept to a friend. However, regarding the types of explanations students preferred their teacher to use, there was no significant preference for either MB or PB explanations.
Educational Studies in Mathematics	"concrete materials" manipulative* mathematic*						
	No limits						

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
2005 Mathematics Education Research Journal	ERIC math* manipulative materials attitude Limits: peer-reviewed	Anne L. Scott	Pre-service teachers' experiences and the influences on their intentions for teaching primary school mathematics	Addressed in this article are two questions: Which teaching strategies and/or practices do pre-service teachers intend using in their teaching of mathematics? and Which experiences influence the development of pre-service teachers' beliefs about teaching and learning mathematics?	Both a quantitative and qualitative study that consists of a survey and semi-structured interviews.	Sample: 163 commencing pre-service teachers and 186 graduating pre-service teachers. No loss	There was little evidence of these people having had experiences that emphasised talking mathematically with their teachers or peers, assessing students' mathematical knowledge, or using calculators. Nevertheless, pre-service teachers held high levels of commitment for some of these practices. The data also indicated that graduating pre-service teachers' intentions or teaching numeracy differed from those commencing the same courses. In particular, more graduating than commencing pre-service teachers intended to find out and build on children's experiences, and they valued teaching their students to use the formal language of mathematics.

Chart 3. Overview of articles

Year of publication	Database	Author	Title	Purpose	Method	Sample	Conclusion
Publisher	Keyword					Loss	Result
	Criteria						
2008 Mathematics Education Research Group of Australasia Inc.	Found in the reference list of <i>Laborativ matematikundervisning - Vad vet vi?</i> (Rystedt & Trygg, 2010)	Stephen Norton & Will Windsor	Student's attitude towards using materials to learn algebra: a year 7 case study	The purpose of this study is to investigate the student perceptions of their own learning and also how materials and material based games help them to understand the processes and structures of algebra at an introductory level.	A case study with both quantitative and qualitative methods consisting of a survey and interviews.	Sample: 24 year 7 pupils Loss: none	Most students believed the learning activities enhanced their understanding of algebra. Due to the pupils' increased competency, the pupils' confidence also increased.

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