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SAFE features for matching fingerprints by neighbourhoods of single minutiae

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Abstract—Symmetry Assessment by Finite Expansion (SAFE) is a novel description of image information by means of Generalized Structure Tensor. It represents orientation data in neighbourhood of key points projected onto the space of harmonic functions creating a geometrically interpretable feature of low dimension. The proposed feature has built in quality metrics reflecting accuracy of the extracted feature and ultimately the quality of the key point. The feature vector is orientation invariant in that it is orientation steerable with low computational cost. We provide experiments on minutia key points of forensic fingerprints to demonstrate its usefulness. Matching is performed based on minutia in regions with high orientation variance, e.g. in proximity of core points. Performance of single matching minutia equals to 20% EER and Rank-20 CMC 69% on the only publicly available annotated forensic fingerprint SD27 database.

Further, we complement SAFE descriptors of orientation maps with SAFE descriptors of frequency features in a similar manner. In case of combined features the performance is improved further to 19% EER and 74% Rank-20 CMC.

I. INTRODUCTION

There exist a number of features that are used in applications of image analysis. e.g. SIFT [1], SURF [2], LBP [3]. These methods have been designed and studied usually with good quality images in mind, whereas we are interested in poor quality images such as forensic fingerprints, here called fingerprints. For example, due to its histograms SIFT features lose spatial distributions of orientation to a certain extent. On the other hand, LBP is bounded by its binary representation when the image is scaled, whereby the description power is restricted.

In the field of fingerprints state-of-the-art matchers usually utilize features that are specially designed for matching constellations of key points, called minutiae, e.g. MCC [4], k-plet [5], Hough transform [6]. For fingerprints, where the number of key points is low, the aforementioned features deliver drastically reduced performance compared to ordinary fingerprints. Currently, a substantial part of the identification work is performed manually by forensic examiners. Main difficulties of fingerprints are small fingerprint area, noisy background and unknown rotation. These complications hinder automatic image processing based features from good performance, which motivates why most of the image processing work is done by human experts. Furthermore, such features

are used only as an addition to the minutia based ones [7], [8] when matching fingerprints.

FingerCode has some common points with the suggested technique in that it also extracts image information around key points [9]. Also, both methods are highly influenced by position of the reference point which can be extracted manually in forensic case scenario. The main difference of the fingercode method is its dimension. Since the feature vector has not a global orientation modelling, the local orientation sampling grows automatically by the size of the spatial area to be described not by the complexity of underlying orientation.

During the recognition process forensic examiner extracts location and direction of key points of fingerprints and then sends them to an Automatic Fingerprint Identification system (AFIS). Algorithms of the AFIS are proprietary and not a common knowledge, but they can be assumed to be using constellation based feature as basis for matching fingerprint images because constellations are what they receive when they process identification requests of fingerprints. Here we propose to extend position and angle features with continuous orientation information of the neighbourhood of the key point.

Section II is dedicated to a summary of preprocessing namely extracting dense orientation and frequency information from the image, even dense frequency is represented as an angle map. In Section III we describe the SAFE process to obtain feature vectors. This is a projection of angle maps on harmonic functions presented in Section III-A and III-B. Rotation invariance of SAFE vectors originating from orientation map is detailed in Section III-C. The paper is concluded with experiments and discussion in Section V.

II. PREPROCESSING: DENSE ORIENTATION AND FREQUENCY MAPS

Feature extraction process has a foregoing step of image preprocessing. For the suggested feature construction, a reasonable quality of orientation and frequency maps is of special significance. This can be provided interactively by forensic expert. However, we did not assume that but used instead a recent automatic method of enhancing orientation together with absolute frequency which was available to us, [10]. It utilizes the theory of Structure Tensor (ST) in logarithmic scale space to estimate absolute frequency and orientation independently. Iterative mutual improvement via enhancement

Fig. 1. Process of feature extraction for a single key point from dense orientation map. *Step 1.* Preprocessing of the original fingerprint image f by means of Linear Structure Tensor (LST). *Step 2.* Extraction of the processed image information at tori in neighbourhood key point (green) in neighbourhood of core (red) and projection of image information around key point into the space of harmonic functions $\Gamma^{\{n, \sigma^2\}}$. *Step 3.* Compensation for rotation invariance

Fig. 2. *Top.* Tenprint and fingerprint images. *Middle.* Corresponding orientation maps in HSV space. Colour/Hue represents the orientation angle, value/intensity represents the quality of estimated orientation. *Bottom.* Frequency maps in HSV colour space with hue/color depicting frequency and value/intensity depicting reliability of estimated frequency for tenprint and fingerprint respectively.

by adaptive Gabor filters leads to maps of improved accuracy of orientation and absolute frequency estimations. Reliability of these orientation maps are supported by high agreement of ridge counts on matched pairs of minutiae on forensic fingerprints (SD27). Preprocessing corresponds to Step 1, Fig. 1 of the feature extraction process where hue represents the angle information as does in Fig. 2.

A. Orientation image

Linear Symmetry Tensor is the mathematical equivalent of ordinary Structure Tensor used for building orientation images. It decomposes the image into directional complex $I_{20}^{(0)}$ and nondirectional real $I_{11}^{(0)}$ parts [11]. This way, we incorporate knowledge of structure tensor into two scalar variables which has clear geometric interpretation in terms of local orientation

and estimation error upper bound

$$LST = (I_{20}^{(0)}, I_{11}^{(0)}) \quad (1)$$

The superscript (0) represents simplest symmetry type which is detailed in Sec. III-A. For all (x, y) such that $ax + by = \text{const}$ any one dimensional function $s(t)$, $s(ax + by)$ will generate a family of isocurves that look like parallel lines. For example, fingerprint pattern can be modelled locally by function $s(t)$ being a sinusoid using a fixed ratio between a, b (the orientation). Then LST is looking for isocurves or parallel lines in (x, y) coordinates and delivers $2 \arctan \frac{b}{a}$, without knowledge of s .

LST is preferred to ordinary structure tensor because of the ability to be decomposed into two visually interpretable information pieces. Complex part of the representation, $I_{20}^{(0)}$, depicts dominant orientation in the neighbourhood of each point which is twice the direction angle of major eigenvector of the Structure Tensor k_{max} along with the error improvement, $\lambda_{max} - \lambda_{min}$, in the magnitude. Real valued $I_{11}^{(0)}$ corresponds to the trace of ordinary structure tensor

$$\begin{aligned} I_{20}^{(0)} &= (\lambda_{max} - \lambda_{min}) e^{i2\angle k_{max}} \\ I_{11}^{(0)} &= \lambda_{max} + \lambda_{min} \end{aligned} \quad (2)$$

In turn, $I_{11}^{(0)}$ quantifies the amount of maximum agreement between the vectors within the analyzed area since the inequality

$$|I_{20}^L| \leq 1, \text{ where } I_{20}^L = I_{20}^{(0)} / I_{11}^{(0)} \quad (3)$$

holds with equality if and only if the orientation fit is error free ($\lambda_{min} = 0$). The latter is the case of perfect linear symmetry when $|I_{20}^{(0)}| = I_{11}^{(0)}$.

This constraint, connects $I_{20}^{(0)}$ and $I_{11}^{(0)}$ in that it allows us to normalize length of vectors of complex valued $I_{20}^{(0)}$ to be guaranteed to be bounded within $[0, 1]$ interval.

We therefore use absolute value of I_{20}^L as a quality measure for orientation estimation, which is given by the argument of I_{20}^L , which equals to twice the angle of the significant eigenvector, eq. (2). In Fig. 2, I_{20}^L is illustrated in colour.

Both parts of the tensor $I_{20}^{(0)}$ and $I_{11}^{(0)}$ are obtained by convolving original image with Gaussians in small scale neighbourhoods whose spatial support is indicated by red circle in Fig. 3, *Left*. Because fingerprint image on small scales is linearly symmetrical, the low magnitudes of $|I_{20}^{(0)}|$ in comparison to $I_{11}^{(0)}$ implies poor orientation fit.

B. Frequency image

The map we are using for extracting frequency information is a remapped absolute frequency. It was established in [10], that frequency and orientation are connected through the trace of structure tensor $I_{11}^{(0)}$ and the size of the Gaussian, σ , used to build $I_{20}^{(0)}$ and $I_{11}^{(0)}$ as

$$\log I_{11}^{(0)} = C - \omega^2 \sigma^2 \quad (4)$$

with C being a constant with respect to σ^2 .

Fig. 3. *Left Black:* Ring shaped filter extraction area. *Left Red:* Filter radii σ for building linear symmetry image. *Right:* Fitting line when obtaining frequency from orientation data.

Then, frequency estimations ω are computed from $I_{11}^{(0)}$ in a scale space of σ^2 by fitting a line in log scale space in total least square sense, Fig. 3, *Right*.

Similarly to estimated orientation map, the frequency map becomes a matrix with complex values I_{20}^F such that $\angle I_{20}^F$ corresponds to twice direction angle of the best fitted line (the tangent of which is ω^2) and $|I_{20}^F|$ become a quality of fitting which includes the corresponding normalization with I_{11} .

Due to the iterative procedure of the algorithm [10] the final frequency image and orientation images as enhanced images converge. Absolute frequency images can thus be displayed by using hue as orientation images since they are represented as angles of the line in Fig. 3. Fig. 2, *Bottom* shows such an image. Because absolute frequency values are in a narrow range for a fingerprint image, colours of the frequency map vary in smaller range than for the orientation image.

Our dense frequency map is thus injectively related to the real frequencies ω via the argument of I_{20}^F whereas the absolute values represents the quality frequency fit.

$$|I_{20}^F| \leq 1 \quad (5)$$

III. SAFE VECTORS

SAFE procedure is applied to orientation or frequency maps consisting of complex values obtained earlier. It can be adapted for any complex image, although orientation and frequency maps ("orientation of frequency" in fact) are preferred due to their representations leading to built in quality estimates and due to direct geometric interpretation in case of orientation maps. SAFE uses harmonic functions, Sec. III-A, for extracting information in ring shaped areas as described in Sec. III-B. Minutia, as well as core has an evident orientation. SAFE vectors of orientations represent features that can be transformed into rotation invariant feature vectors, Sec. III-C. An element of SAFE vector is related to a predefined torus shaped area, Step 2, and equals to the projection of the area to a harmonic basis function, Step 3, of Fig. 1.

A. Harmonic functions

LST (1) represents a family of isocurves having same direction in Cartesian coordinates (x, y) , but the concept can be extended to fairly general coordinate transformation ξ, η leading to the notion of Generalized Structure Tensor (GST) [12]

$$\begin{aligned} GST &= (I_{20}^{(n)}, I_{11}^{(n)}) = \\ &= (\Gamma^{\{n, \sigma_{out}^2\}} * (\Gamma^{\{1, \sigma_{in}^2\}} * f)^2, |\Gamma^{\{n, \sigma_{out}^2\}}| * |\Gamma^{\{1, \sigma_{in}^2\}} * f|^2) \end{aligned} \quad (6)$$

Fig. 4. Sample patterns of harmonic functions generated by means of GST, [13].

with the $\Gamma^{\{n, \sigma^2\}}$ being n 's symmetry derivative of a Gaussian function with variance σ^2 when $n \neq 0$:

$$\Gamma^{\{n, \sigma^2\}} = r^n \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} e^{in\varphi}, \text{ with } r = |x + iy|. \quad (7)$$

In case of $n = 0$ GST transforms into LST with the Γ being Gaussian instead of its derivative.

On the other hand, family of harmonic functions is a set of locally orthogonal analytical functions ξ, η such that function $s(a\xi + b\eta)$ generates a family of isocurves, Fig. 4, if $\xi(x, y) + i\eta(x, y) = g(x + iy)$.

For example, choosing the one dimensional function $s(t)$ as sinusoidal and choosing $g(z)$ yields $\xi = \sqrt{x^2 + y^2} \cos(\frac{1}{2} \arctan \frac{y}{x})$ and $\eta = \sqrt{x^2 + y^2} \sin(\frac{1}{2} \arctan \frac{y}{x})$ generating parabolic pattern, via $s(a\xi + b\eta)$. GST has the ability to detect harmonic functions according to the value of parameter n , e.g. parabolic pattern is generated by GST with $n = -1$ by convolving a corresponding complex detection filter with the orientation image. This procedure generates a complex pair $(I_{20}^{(n)}, I_{11}^{(n)})$ in complete analogy with (1)-(3).

B. Feature extraction

Feature extraction is performed by means of GST, eq. (6), with the Gaussian derivative function defining the area of extraction. In the given representation of Gaussian derivative, eq. (7), we intentionally used polar complex representation. Parameter n of the real part defines the size of the area to extract feature from, whereas n of the complex part is the one responsible for generating pattern of the desired type (e.g. parabolic, spiral, linear, etc.). To highlight that fact and in order to be able to control size of the analysis area independently of the pattern, we will use the following representation instead

$$\psi_{kn} = r_k^{\mu_k} e^{-\frac{r_k^2}{2\sigma_k^2}} e^{-in\varphi} / \kappa_k = |\psi_{kn}| e^{-in\varphi} \quad (8)$$

with κ_k being the norm of ψ_{mn} , i.e. $\langle \psi_{kn}, \psi_{kn} \rangle = \kappa_k^2$. Here $k = 1 : N_f$ with N_f being the number of different rings to be analyzed and $n = 1 : N_h$ with N_h being the highest complexity by which each ring will be analyzed. N_h represents thus harmonic basis on which to project the ring of complex image (orientation or frequency data).

With this updated representation we can control shape, width and position of the area. We can design sets of ring shaped filters with exponentially growing radii so that Gaussians capturing image area to be analyzed will overlap each other at any predetermined amount. It is straight forwards to show that

$$r_k = \sqrt{\mu_k} \sigma_k \quad (9)$$

¹ Soon the two n 's are decoupled and what is the exponent of r is called μ_k and what in front of φ is called n , which is also the symmetry order, remains as is

where r_k represents the radius at which ψ_{kn} is maximum. We fix r_k to be in a desired interval $[r_{min}, r_{max}]$ and sample other radii log-equidistantly. We then fix the parameter μ_k according to our desire for filters to have constant overlapping area (to be as small as ϵ) between consecutive rings. Here we have chosen $\mu_k = 20$ yielding the overlap ϵ .

GST is comprised of two, consequently applied complex convolution to the image. Gaussian derivatives form ring shaped areas for information analysis, Fig. 3, *Black*. Width of the ring is controlled by μ_k . To select the area of the image within tori area corresponding to the filter to be applied, we simply use the magnitude of the filter

$$h_k = I_{20}^L \cdot |\psi_{kn}| \quad (10)$$

This explicit extraction is a necessity to preserve the quality essence of the projections. Similarly to Linear Symmetry Image ($I_{20}^{(0)}$ and $I_{11}^{(0)}$), we convolve filters with the image

$$I_{20}^{(k,n)} = c_{kn} = \langle \psi_{kn}, h_k \rangle = \langle |\psi_{kn}|^2 e^{-in\varphi}, I_{20}^L \rangle \quad (11)$$

$$I_{11}^{(k,n)} = e_{kn} = \langle |\psi_{kn}|, |h_k| \rangle = \langle |\psi_{kn}|^2, |I_{20}^L| \rangle. \quad (12)$$

Equation (11) results in complex feature vector with angle of it depicting a particular member of the harmonic function family it was mapped into, and absolute value describing quality of estimated member, therefore allowing to introduce novel feature **SAFE**:

$$SAFE_{kn} = c_{kn}/e_{kn} \in \mathbb{C}, |SAFE_{kn}| \leq 1. \quad (13)$$

For example different members of the family, e.g. $n = -1$ are parabolas rotated in different amounts, Fig. 4, $z^{0.5}$.

C. Rotation invariance and quality of the feature

The argument of $SAFE_{kn}$ when extracted from orientation map is related to the direction of isocurves and highly depends on the global orientation of the image. If there is a direction associated with key point, SAFE vector of orientation offers rotation invariance. In case of fingerprints, both minutia and core has clear direction, therefore we can alter $SAFE_{kn}$ to be rotation invariant with insignificant computational cost without rotating the original images.

Assume that we wish to obtain $SAFE_{kn}$ but for the original image rotated with amount φ' . Then it can be shown that the corresponding feature rotation is achieved by a simple complex multiplication from original SAFE vector without image rotation

$$SAFE'_{kn} = e^{2i(n+2)\varphi'} SAFE_{kn} \quad (14)$$

Together with rotation invariance key points have built in quality estimate which is implicitly used in the matching stage. However, it can also be extracted if needed for evaluating the quality of the key point similar to what forensic examiners do when they match manually.

According to triangle inequality for any family of harmonic functions the following inequality holds

$$|I_{20}^{(k,n)}| \leq I_{11}^{(k,n)} \quad (15)$$

Geometrical interpretation for $n = 0$ was discussed earlier in Sec. II. Similarly, for other values $I_{20}^{(k,n)}$ specifies the direction of the harmonic basis pattern defined by n . In case of spirals, $n = -2$, $I_{20}^{(k,n)}$ defines rather the twist of the spiral than global direction.

Our $I_{20}^{(k,n)}$ are therefore normalized, eq. (13), and one can use their magnitude as a quality measure

$$|I_{20}^{(k,n)}|/I_{11}^{(k,n)} = |SAFE_{kn}| \leq 1. \quad (16)$$

The quality measure reflects model fit and this is influenced by the quality of the data.

IV. MATCHING IN COMPLEX SPACE

We base matching on triangle inequality for complex valued matching score, MS

$$MS = \frac{\langle SAFE^r, SAFE^t \rangle}{\langle |SAFE^r|, |SAFE^t| \rangle} \in \mathbb{C} \quad (17)$$

where $SAFE^r$ and $SAFE^t$ are reference and test complex feature vectors respectively. Here $\angle MS$ is the angle between the reference and test vectors and $|MS|$ is a reliability of the estimated distance which depends on the quality of data and amount of data used for feature construction. The angle $\angle MS = 0$ represents the highest similarity whereas $\angle MS = \pi$ represents the highest dissimilarity between the two vectors. Accordingly, the real part of MS is the score can be used for decision making.

$$\hat{MS} = Re(MS) = |MS| \cos(\angle MS) \in [-1, 1] \quad (18)$$

V. EXPERIMENTS OF FINGERMARKS

We provide experiments on the NIST Special Database 27 (SD27) of forensic fingerprints, [14]. Database consist of 258 pairs of fingerprint images, one obtained at crime scene and one collected in supervised manner, at different points of time.

SAFE feature applied to core and loop singular points has shown high level of reliability and distinctiveness [15]. Here we test SAFE at individual minutiae, in order to show that existence of singular points is not a requirement for the feature vector to be useful. At the same time the goal is to indicate that all minutia in the singular point area are equally valuable for describing orientation change. We have selected 1-2 points from every image in areas of significant orientation change (in neighbourhood of core or deltas if present) resulting in 320 clients and 50,978 impostors. Because we are testing the descriptive power of features at a single point, without using the constellation of minutiae, such minutia are good candidates for testing descriptive power [16]. Annotations of the expert for location and direction of the minutiae were used when testing our features. All other steps are performed in 'lights out' manner.

We set number of filters $N_f = 9$ with sampling interval $[r_{min}; r_{max}] = [2; 97]$ to project onto $N_h = 9$ symmetry basis, without a priori information about fingermark applications. Experimental results showed that feature is stable in the areas of higher radii of filters, therefore we select only 3 highest tori represented by the feature [27, 47, 63].

Fig. 5. Evidence on one minutia closest to core of the fingerprint. *Top*. Matching SAFE feature extracted from orientation images. *Middle*. Matching SAFE feature extracted from frequency images. *Bottom*. Combination of different source features

Fig. 6. *Left*. Erroneously matched fingerprint based on orientation based feature. Frequency has improved matching score substantially. Matching is performed for minutia 13. *Right*. High confidence match for minutia near the cores.

SAFE descriptors were used to describe neighbourhoods of orientation and frequency dense maps separately Fig. 5, both showing competitive performance, EER 20% and 28%, rank-20 CMC 69% and 45% respectively. Due to low expressive power of the confidence of frequency maps (as they were obtained from enhanced images), frequency obtained SAFE descriptor is less narrative. Joining together descriptors we can improve performance further to 19% EER and 74% rank-20.

It is difficult to relate descriptors to existing forensic fingerprint features as they use incomparably more manual inputs (manually extracted minutiae) or sufficiently more key points (minutiae constellations), at the same time information is left unused. We have tested the feature on the only available forensic fingerprint database in order to show the quality of extracted image-based feature; therefore number of images significantly differs with published studies on non forensic fingerprints. The goal of the paper is to present the new concept of feature extraction. Further experiments on multiple minutiae are required for benchmarking present work. The observed EER using a publicly available (source code) minutiae-only matcher shows 36% EER [16] performance on an ideal set (and 6% on a matched set - which is an unrealistic scenario), which is significantly poorer than the achievement of the suggested method.

VI. CONCLUSION

We tested SAFE feature for matching minutiae of the forensic fingerprints. SAFE feature is image-based feature therefore it was important to show its performance on low quality and complicated background forensic images. We applied SAFE descriptor to express orientation and frequency image informa-

tion. Both maps has shown independently reliable performance on SD27 annotated forensic databased reaching 20% (69% rank-20 CMC) and 28% EER (45% rank-20 CMC) respectively which is comparable to constellation based matchers reported on this database (e.g. [6]) considering that we did not use minutia constellation information. Combination of two features improve matching further to 19% EER (74% rank-20 CMC).

SAFE features are orientation steerable, in other words, there is no need to rotate the reference image towards to the test-image, since descriptors can be compensated for rotation by complex multiplication.

Future work includes testing feature as complementary to state-of-the-art features used to match forensic fingerprints. The latter are essentially minutia locations, and directions. Additionally we plan to apply feature to images of different source, for example, forensic shoe prints or images of irises.

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