Mathematics teachers’ strategies for supporting students’ metacognitive development: Has theory been realized in practice?
Abstract

Title: Mathematics teachers’ strategies for supporting students’ metacognitive development: Has theory been realized in practice?
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The purpose of this study is to investigate (1) how mathematics instructors develop their students’ metacognitive abilities concretely within mathematics instruction and (2) whether these teachers feel adequately prepared to develop their students’ metacognitive abilities. Qualitative email interviews with credentialed secondary school mathematics teachers in Sweden were used. Analysis of the participants’ interview responses indicate that the participants reported a limited use of the metacognitive teaching strategies described in the research. Although teacher responses indicated stress, frustration, and irritation and their responses indicated limited proficiency in their intuitive declarative metacognitive knowledge of thinking skills, whether or not teachers feel adequately prepared to develop their students’ metacognitive abilities cannot be completely answered by this study.
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1 Introduction

Metacognition, thinking about one’s own thinking (Flavell, 1971), has become widely recognized as a strong indicator of students’ success within school and particularly mathematics. Although studies have been conducted to measure students’ metacognitive abilities and their success within school as well as to define different techniques for teaching and assessing metacognition, the question must be asked if these relatively new techniques are effectively put into practice in the classroom or if more work needs to be done to put theory into practice?

This study has both a personal and theoretical relevance. On a personal level, I have previously worked in California as a math teacher at the high school level and have been required to complete my education in order to be a credentialed mathematics teacher in Sweden. During my Swedish teacher education I have learned a new way of looking at how we teach and assess students from a more qualitative perspective. My education in Sweden has deepened my understanding of the theoretical background of the Swedish educational perspective, but has left me feeling less than completely prepared to meet my students when it comes to how we, practically, can develop a deeper quality of understanding and thinking in our students. To become more professionally prepared before I come out into the work place, I have chosen to research how I can develop my students’ metacognitive abilities. During my research, I have found practical solutions to both instruct and assess students metacognitively. This has left me wondering two things:

1. If the answers I have found are readily available in the current literature, are mathematics teachers actively employing it in their practice?
2. Are there other teachers who also feel less than completely prepared to meet their students’ metacognitive needs?

In investigating the answers to the above questions, hopefully I can inspire mathematics teachers to reflect over their own metacognitive teaching practices, and, if other teachers feel less than completely prepared in this regard, to inspire teacher education programs to reflect over how teachers are trained and educated to develop students’ metacognition.

From a theoretical perspective, the Swedish government has written new goals within the area of mathematics at the upper secondary school level (here I choose to use Swedish secondary education to refer to the level of school in that most closely resembles high school in the US both with respect to age and content: gymnasieskolan) that make it mandatory for teachers to assess students’ ability to guide and reflect upon their own learning. In order for students to achieve the highest possible grades they must now be able to “utvärdera med nyanserade omdömen resultatets rimlighet samt valda modeller, strategier, metoder och alternativ till dem.” This exact same learning goal is reiterated for every mathematics course within Swedish secondary education (Skolverket, 2011). No longer is it sufficient for students to arrive at a correct mathematical solution, but they must now be able to determine the reasonableness of their solutions, judge the efficacy of their chosen approach, and consider other possible strategies. These tasks require that students actively reflect upon their work, a characteristic of metacognition.
Swedish students have lost ground in mathematics when compared to other countries on measures such as PISA in recent years and the government has recently invested over 300 million Swedish crowns to investigate the reasons behind the decline and to develop and improve mathematics instruction (Skolverket, 2011). Can improving our understanding of and practical abilities in teaching mathematics more metacognitively narrow this gap?

Initially the intention of this research was to define effective practices that mathematics instructors could use to assess and develop metacognitive abilities in their students as a means to improving mathematical understanding and performance. When an analysis of the theoretical background and prior research was conducted, I found that those studies have already been done. As recently as 2008 Enroth & Nylén concluded that when they studied how teachers can use a metacognitive teaching approach:

"Inom forskningen har en allt större vikt lagts på metakognition i undervisningen, och hur den kan bidra till en ökad förståelse inom matematik. Däremot är det inte helt självklart att forskningen införlivas i lärokulturen på skolorna. Den uppfattning vi fått efter arbetet med denna undersökning är att även om metakognition förekommer frekvent i aktuell studielitteratur på Universitetet, verkar ämnet ändå vara förvånansvärt okänt hos både lärare och lärarstudenter. Vi upplever att kunskap om metakognition är ovanligt i skolan, och att ett metakognitivt arbetssätt därför inte används i någon större omfattning (p. 35)."

That is to say that although metacognition is an essential component to successful teaching of mathematics and is widespread within the current literature, the researchers experienced that both teachers and teacher students were surprisingly lacking in their knowledge and awareness of a metacognitive teaching approach and therefore did not witness its consistent use.

Additionally, Olsson (2011) found that when studying teachers’ attitudes towards metacognition that, across all subject areas, teachers scored lowest in their ability to train their students’ metacognitive understanding when compared to how teachers themselves understand metacognition, teachers’ attitudes towards metacognition, and the importance teachers give metacognition during assessment and grading. Olsson also found that mathematics teachers scored lower than other subject matter teachers when looking at their ability to train students in metacognition. More details on Olsson’s study can be found later in the section titled “prior research” (section 3.2).

As a result this study seeks to find how, through the use of email interviews analyzed qualitatively, mathematics teachers work to develop metacognition in their students and whether they feel adequately prepared to meet the new learning goals set forth for mathematics education in Sweden.

1.1 Purpose

In order to study how mathematics instructors develop their students’ metacognitive abilities concretely within mathematics instruction and to study whether these teachers feel adequately prepared to develop their students’ metacognitive abilities, the following questions will guide this study:

1. Does teacher practice match the current research on metacognitive
2. Do mathematics teachers feel a need to increase their level of expertise when developing their students’ metacognitive abilities?

2 Theoretical Background

2.1 Definitions

2.1.1 Metacognition and metacognitive knowledge

Flavell (1971) first defined metacognition as thinking about one’s own thinking. Flavell thought that, in order to develop one’s metacognition, one must first know what to monitor (metacognitive knowledge). Flavell states that metacognitive knowledge is gained through experience. The individual then sets goals to achieve understanding and uses strategies to monitor his/her progress toward learning goals. Similar definitions have also been proposed. Dysthe, Hertzberg & Løkensgard (2002) mean that metacognition is not only being aware of the content of what is learned, but also how it is being learned. Here it is possible to draw parallels between what Flavell calls monitoring and what Dysthe et. al. call awareness. If one can monitor progress of thought and learning one must be aware of it.

The definition of metacognition also includes another component: regulation. Hacker et. al. (1998) described metacognition as knowledge about one’s own knowledge and the ability to consciously regulate it. Schoenfeld (1992) described metacognition as knowledge about and regulation of cognition. Veenman et. al. (2006) defined metacognition as the knowledge and regulation of one’s cognition during learning. Parallels can be drawn between the ideas of regulation and control. To regulate thinking and learning one must have control over it. That can only first be achieved by having awareness of it.

Researchers have also illuminated the difference between cognition and metacognition. Garofalo and Lester (1984) simplify cognition as “involved in doing, whereas metacognition is involved in choosing and planning what to do as well as monitoring what is being done” (p. 163). Understanding metacognition can be difficult because it is used to refer to both knowledge about and regulation of cognition (Brown, 1987 and Schoenfeld, 1990). Schoenfeld means that the difficulty in understanding metacognition can lie in the fact that cognition refers to the processes of the mind which are not only regulated by metacognition but also include metacognition.

Wilson (1997, p5) has synthesized the definition of metacognition as “the awareness individuals have of their own thinking and their ability to evaluate and regulate their own thinking.” Here Wilson’s choice of words can be closely compared to previous definitions. When one evaluates their own thinking they are aware of their thoughts and their quality. When one regulates their thinking they are actively involved in making judgments about the efficacy of their thought process. One can draw the conclusion that thinking and learning can be improved once one has become aware of the processes of learning and thinking, learned to monitor and regulate them, and deliberately taken control over them.
We must be able to improve educational outcomes by teaching learners how to learn. If a teacher is to be engaged in improving the quality of what students are learning it is essential that instruction focus not merely on the product that is expected, but also on the process that leads to the desired product. Metacognition can be viewed as the internal process that can lead to, when active improvement is sought, a better quality of understanding.

2.1.2 Metacognition, self-regulation, and reciprocal determinism

The term metacognition has given rise to other similar terms, one being self-regulation. Baker and Brown (1984) separated the metacognition into knowledge about cognition (monitoring) and self-regulatory mechanisms for checking the outcome, planning, monitoring effectiveness, testing, revising, and evaluating strategies. Baker and Brown have differentiated between monitoring (awareness) and regulation (evaluation and judgment making).

Although self-regulation grew out of definitions of metacognition, they should not be considered the same term. Dinsmore et al. (2008) suggest that self-regulation refers to how the environment acts as a stimulus of the individual’s awareness and regulation, whereas metacognition emphasizes the individual’s mind as the “trigger for subsequent judgments or evaluations.” Dinsmore differentiates self-regulation and metacognition based upon the distinction of internal vs. external, but they must work hand in hand. It should be considered that the two are not necessarily mutually exclusive. If cognition and metacognition can be considered human behaviors we must take into account the interplay between internal and external factors that affect behavior. ” Bandura (1977) raised the importance of reciprocal determinism where human functioning is based on an interaction between person, behavior, and environment reasoning that self-regulation cannot occur without interaction with the environment” (in Lajoie, 2008 p. 470). It becomes clear, however, that both the individual and the environment (which includes, among other things, fellow students and the teacher when looking at a classroom environment) must be taken into account when looking at the monitoring and regulation of thought.

2.2 Importance of metacognition for school success

A student’s level of success in school has been linked to his/her level of metacognition by a number of researchers. The following researchers have linked metacognition to the improvement of specific school-related tasks, cognition, and motivation.

Flavell (1985) identified metacognition’s importance in many tasks including oral communication, reading comprehension, writing, attention, memory, problem solving, and self-control. Both Imsen (2000) and Hartman (2001) have discussed the importance the awareness of one’s thoughts can have for the improvement of thought processes. Metacognition has also been linked to improved student motivation and thereby improved performance within school (Pramling, 1992 and Imsen, 2000).

If metacognition is a deciding factor for school success in a wide variety of subject areas and we want students to be successful, then it is something that we, as teachers ought to be teaching. Teaching metacognition can improve the ability to generalize
specific knowledge and increase insight to one’s own learning process (Hartman, 2001). A student must be able to take strategies learned in one situation and apply them in others. Through self-assessment and planning students become capable of taking strategies learned in one learning situation and apply them to others. Additionally, if a student learns what strategies and methods lead to successful learning of content, the student becomes capable of learning more than students lacking this insight.

As stated earlier the term self-regulation has grown out of the research on metacognition. Zimmerman (2002) states that improved self-regulatory skills improve academic performance. Zimmerman’s (2001) three-phase cyclical model of self-regulation reinforces the importance of metacognitive processes for school success. In the first phase, forethought, beliefs about one’s own abilities (self-confidence judgments) prepare the student for learning. In the second phase, performance, students implement learning strategies such as metacognitive monitoring. In the third stage, self-reflection, students self-evaluate progress and adjust as necessary for later learning cycles. Schunk (1996) noted that students who adequately self-reflect over their learning process gain a higher level of competence which in turn increases their self-confidence beliefs in the forethought phase, thus leading to improved learning results in future learning situations and completing the cycle.

As a teacher of mathematics I have encountered a number of students who have tried to sabotage themselves before they have even walked into the classroom for the first time. It has been my experience that before I can expect such a student to learn new concepts and strategies within mathematics, I must first help the student change their perspective from “I can’t” to “I see what I can, now let’s see what else I can.” If, as Schunk (1996) noted, helping students to self-regulate accurately results in a greater sense of capability then we, as mathematics teachers, have a powerful tool to reverse self-sabotage. What other positive effects can working with metacognition have on learning mathematics?

### 2.3 Metacognition and success in mathematics

Not only has metacognition been linked with general success within school, but research has also shown it to be particularly important within mathematics. Metacognitive ability has been shown to play a role in problem solving, mathematical performance, accurate self-efficacy judgments, and motivation.

One can argue that success in mathematics depends upon one’s ability to solve problems. Both Flavell (1985) and Goos (1993) indicate the importance metacognition plays in problem solving. Schoenfeld (1985) states that students adept at problem solving engage in an internal dialogue where they constantly evaluate the evolution of their solutions and not just the solutions themselves. The students’ cognitive and metacognitive processes play a role in problem solving (Schoenfeld, 1985): students who are more proficient problem solvers are not just engaged in solving the problem (what Garofalo & Lester (1984) called “doing”) but also in regulating their own processes leading to the solution (“choosing, planning, and monitoring” for Garofalo and Lester).
Kramarski & Mevarech (2003) found that students who receive metacognitive training outperform students on mathematical tasks who do not receive such training. Those who received metacognitive training were able to find multiple arguments for their solutions more often than those students who did not receive the training. In addition, Kramarski & Mevarech (2003) found that those who received metacognitive training in groups outperformed those who trained individually. These results indicate that a cooperative learning environment is advantageous in the development of metacognitive abilities when assessing their impact on success in mathematics.

In can be argued that students who believe they can achieve success in mathematics have a better chance at succeeding than those lacking that same self-assuredness. We have already seen the role that accurate self-monitoring plays in improving students’ self-monitoring cycle and thereby affecting the forethought phase in subsequent cycles (Schunk, 1996 and Zimmerman, 2001). “Self-efficacy is the belief in one’s capability to organize and perform a set of activities necessary to complete a task at a specified level of competency” (Bandura, 1997). Ramdass & Zimmerman (2008) have indicated that students who have inaccurate self-efficacy beliefs may avoid challenging tasks such as advanced mathematics courses.

Students who have a high motivation for mathematics may, merely due to their desire to learn mathematics, have more success within the subject. Enroth & Nylén (2008) found, when studying the relationship between students’ metacognitive abilities and their level of motivation within mathematics, a connection between (1) students’ metacognitive ability and how enjoyable students see mathematics, (2) the students’ level of self-confidence within mathematics, and (3) how important mathematics is to students.

If a well-developed metacognitive ability is connected to performance within mathematics, can the opposite be said? Do low-performing mathematics students have a more limited metacognitive ability? Munro (1993) has stated that students who struggle with mathematics lack cognitive or metacognitive strategies to employ. Ramdass & Zimmerman (2008) found that not only is inaccurate self-reflection detrimental to students success within mathematics (particularly when studying alone), but that students with “unrealistic self-efficacy beliefs” may choose to avoid mathematics and other challenging academic subjects based upon these beliefs and not a lack of ability.

It is evident that metacognitive ability or lack there-of is essential for developing capable, motivated students of mathematics. If so, how can we, as teachers, help our students develop this ability?

2.4 Teaching metacognition

Self-assessment is the process by which students make judgments about their own learning (Boud, 1991) and is a “means to develop metacognition” (Wilson, 1997). It is easy to have a preconception of not only self-assessment but also assessment in general. Assessment can not only be formal (written quizzes or tests), but can also be informal. Assessment, both formal and informal, written and verbal should be seen as an integral component to learning. But in what ways can we engage our students in self-assessment and thereby help to develop their metacognitive abilities? Is there
evidence of the effectiveness of other strategies teachers can implement to develop metacognition in their students? In this section I will discuss nine different methods teachers can use to help in this development:

1. Questions and questioning strategies
2. Group discussions
3. Thinks-alouds
4. Summarizing
5. Debriefing
6. Journal writing
7. Guiding self-efficacy
8. Accuracy training
9. Efficient allocation of study time

2.4.1 Questions and questioning strategies

The first strategy teachers can employ is how they communicate in the classroom with students. Dimenäs (1995) has said that although questions and answers are a dominant means of communication in classrooms, teachers must be cognizant to give room and status for students’ own questions and reflections. It could be easily assumed that teachers plan the questions they intend to pose to students without thinking of the time and space that need be allotted for the questions created by their pupils.

This should not presuppose that teacher posed questions are less important in comparison, but the type of questions teachers pose to students is also critical in developing metacognition. Pashler et al. (2007) have described seven different points in Organizing instruction and Study to Improve Student Learning, two of which can be seen to support metacognition. One of these that supports metacognition is that teachers “use deep, explanatory questions”. Why and how questions have been recommended in order to cause the student to reflect (Hemberg, 1995) and to determine causes, explain reasoning, and draw conclusions (Dimenäs, 1995). Dimenäs (1995) goes on to say that these questioning techniques have other advantages in the classroom: they move the classroom from teacher-centered to student-centered and they send a message to students about the teacher’s ideas of learning and knowledge. If we want students to achieve deeper understanding, than we ought to create an environment where their thoughts and thought processes are the focus and where classroom practice implicitly values the depth we hope to achieve.

In addition to why and how questions, research shows another type of questioning strategy that can improve metacognition. Sjöström (1998) suggests the use of metacognitive questions including: “What have we been trying to accomplish during this lesson?” “What do we hope to achieve with these exercises?” “What have you learned from this discussion?” “How did you learn it?” “How do you know what you know?” and “How does your teacher know what you know?” These questions cause the students not only to reflect on the content they have learned but also the manner in which they have learned it and whether they have indeed learned it.

Kernell (2002) prescribes another questioning strategy for teachers to employ that can help students understand what we expect in them during instruction. These questions can include “What is the purpose and learning goals we are focusing on?” and “Why
have we chosen this approach and not another?” Kernell (2002) also suggests that we ask ourselves questions about our own instruction: “What are they going to be able to do that they can’t do now?” and “What do the students think that they have developed that I may not have been aware of?” These questions serve not only to help the teacher develop metacognition in their students, but also engage the teachers themselves in the process of developing metacognitively.

### 2.4.2 Group Discussions

According to Vygotsky’s sociocultural theory of human development, people learn from one another primarily through language. Vygotsky’s theory highlights the importance of human social interaction in our lifelong developmental process. It is Vygotsky’s sociocultural theory of human development that can be seen as a theoretical basis for the use of group discussions to meet learning goals. Group discussions are also relevant to the development of metacognition.

In *Informed strategies for learning: a program to improve children’s reading awareness and comprehension* (Paris, 1984) found that direct instruction can promote metacognitive development. Paris also highlighted four strategies for the teaching of metacognition:

1. Awareness of strategies
2. Accepting metacognition’s importance
3. Importance of giving students time to practice strategies they have learned
4. Importance of group discussions

Kahveci & Imamoglu (2007) suggest that, when investigating different types of interactions in mathematics classrooms, there are a number of teaching strategies that provide positive results. These include using multiple representations, creating contexts for mathematical argumentation, encouraging student participation in classroom discussions, expecting mathematical reasoning rather than correct answers, and designing tasks to promote mathematical reasoning and metacognition.

### 2.4.3 Think-alouds

Think-alouds are another method that can be beneficial to metacognitive development (Bondy, 1984). This teaching method involves the teacher sharing their thoughts about how they “tackle unfamiliar tasks and problem solving situations” verbally with students. Teachers can share with their students how they discover the aspects of a particular problem that they find difficult and the strategies they choose to employ to overcome these difficulties. In a mathematical setting this concretization by teachers can be useful in showing how to call upon strategies used in similar problem solving situations previously.

Bondy (1984) also suggests that teachers share their metacognitive thoughts with their students. These can include “estimating task difficulty, identifying goals, choosing strategies, identifying a sequence of steps, and planning for evaluation.” Bondy (1984) states that this type of modeling helps the students to formulate their own thoughts and stresses the learning process over the learning product.
2.4.4 Summarizing

“When the learner monitors their thinking, makes judgments about their own thinking or consciously decides to act upon their reflections, they have been metacognitive” (Wilson, 1997 p. 6). In order to organize student reflections and develop students’ monitoring skills the literature suggests that students summarize their learning.

Bondy (1984) suggests that students be explicitly taught how to summarize what they have learned because summarizing clarifies for the student how well they have understood the learning goals.

2.4.5 Debriefing

Leat & Lin (2003) used debriefing as a technique for improving student learning. Debriefing is described as a group or whole class discussion that is done after learning activities in order to encourage students to consciously explore and broaden their learning. In the discussion students are asked to reflect upon what they learned that day and what the teacher did to help them learn. These reflections help students monitor both what they learned (product) and how they learned it (process).

2.4.6 Journal writing


Although it could be argued that a teacher using deep, exploratory questions and metacognitive questions as well as debriefing strategies has already encouraged their students to summarize and reflect over their thinking and learning, the use of journals causes the student to reflect and summarize in a much more formal manner. Putting one’s thoughts on paper may be more permanent and thus used for future reflections including preparation for summative assessments.

2.4.7 Guiding self-efficacy

In the previous section we see how students can be supported in developing their self-monitoring skills. Here I take a look at how teachers can support students to develop their ability to “accurately make judgments about their own thinking and consciously act upon those judgments” (Wilson, 1997 p. 6).

“Classroom practice must not only cultivate the knowledge to succeed, but should nurture the belief that one can succeed “ (Ramdass & Zimmerman, 2008 p. 37). If one does not believe they can succeed in any endeavor they are very likely to do just that. We need to make sure that we not only give the students the tools to succeed and the knowledge necessary to utilize those tools in the best possible way, we must also make certain that the students believe that they can master the tools we give them. The research provides a number of ways in which we can guide our students’ self-efficacy.
Theobald (2006) discusses the responsibility teachers have to build up their students’ accurate self-efficacy beliefs and suggests that teachers begin new areas of study by asking students to write down everything they know about that topic. This strategy focuses the student on what they do know rather than what they don’t know. It can be assumed that many teachers, particularly in mathematics, use different diagnostics to determine what is to be taught. These diagnostics, or pre-tests, can be assumed to highlight what students do not know. It cannot be assumed that we should ignore or downplay the use of such diagnostics, but it should be complemented with strategies that help the students become aware of what they already can do.

Bandura (2006) and Chen (2003) both suggest the use of metacognitive questioning to help guide students self-efficacy. Bandura suggests asking students about their capability in completing mathematics problems and Chen suggests asking students to rate how certain they are in their solutions.

2.4.8 Accuracy training

Pashler et al. (2007) suggest that, particularly when reviewing a topic, that students rate their own ability to understand and solve problems. After students rate their problems’ difficulty level they return those problems rated most difficult. This process is repeated until only a small number of problems each student rates as most difficult remains. Oakes & Starr (2008) state that Pashler et al.’s approach is a good method for developing students’ metacognitive abilities and making them more aware of what they do and do not understand. Ramdass & Zimmerman (2008) also advocate the use of accuracy training techniques. They suggest that teachers show students how well they gauged their own capabilities. By showing this, students become more adept at realistically assessing their capabilities.

2.4.9 Efficient allocation of study time

Pashler et al. (2007) suggests that teachers use assessments (tests and quizzes) to identify for students what they still need to learn. Here it should be noted that Pashler et al. encourages the use of tests and quizzes in a formative rather than summative purpose. If the students are taught to be more accurate in their self-efficacy judgments and learn that they are capable of altering these judgments based upon continued work, they will not only become more confident in what they know, but also more aware of the things they don’t know, thereby allocating their study time more effectively.

3 Prior Research

3.1 A metacognitive teaching approach and its effect on students' motivation towards mathematics

Josenfin Enroth och Matilda Nylén (2008) in their thesis *Att skapa motivation i det matematiska lärandet* conducted a study of how a metacognitive teaching method could effect how students at two secondary schools in and around Stockholm, Sweden experienced mathematics.

Students were given a problem and asked to work for a short period of time individually on the problem and then followed that by working in self-selected small
groups of three to four students to discuss their approaches to the problem. Students were encouraged to not erase anything they had thought of in solving the problems due to the fact that the researchers had explained that it was the students thought processes behind their strategies that were of interest, not necessarily whether or not students could achieve a correct final answer.

During the individual problem-solving phase of the process, students were asked to write their answers to three metacognitive questions:

1. Which strategies are useful?
2. Why are they useful?
3. How can they be used to find a solution? (own translation)

These three questions were later analyzed to determine the students’ “metacognitive competence” which the authors describe as their ability to think about their learning.

During the group problem-solving phase, students were encouraged to ask each other questions about their strategies, offer any tips they thought could be helpful to others, and to assist each other in answering the three metacognitive questions above. At the conclusion of the group phase students were asked to answer the question, “Do you think that the way in which we have worked can help to develop your understanding of mathematics? Explain why!” (own translation).

Students were also asked during the study to rate how fun/boring, how important/unimportant, and how easy/difficult they think mathematics is for them. Students’ answers to these questions were then translated into a ten-point scale and analyzed quantitatively.

Results indicated that:

1. There were more students with less developed “metacognitive competence” than more developed “metacognitive competence.”
2. Students with more developed “metacognitive competence” found mathematics to be more fun and meaningful.
3. Students with more developed “metacognitive competence” reported greater self-esteem within mathematics.
4. The group discussion phase was more meaningful for those with a more developed “metacognitive competence.”
5. Those that thought the group discussion was more meaningful had a more positive attitude to working metacognitively.
6. Both students with less developed and more developed “metacognitive competence” reported that they viewed mathematics as important.
7. Students of all levels of “metacognitive competence” reported that reasoning with themselves and others can help them improve their understanding of their own thoughts and of mathematics.

Although all of the above results support the link between metacognition and self-esteem, motivation, and performance on problem solving tasks, it was the authors’ final conclusion that can be seen as a strong impetus for this study (as stated previously):

Inom forskningen har en allt större vikt lagts på metakognition i undervisningen, och hur den kan bidra till en ökad förståelse inom matematik. Däremot är det inte helt
That is to say that although metacognition is an essential component to successful teaching of mathematics and is widespread within the current literature, the researchers experienced that both teachers and teacher students were surprisingly lacking in their knowledge and awareness of a metacognitive teaching approach and therefore did not witness its consistent use.

Enroth & Nylén’s findings and subsequent commentary has led me to the first of the questions guiding this research: Does teacher practice match the current research on metacognitive mathematics instruction?

3.2 Teachers’ attitudes towards metacognition and self reported metacognitive practice

Kajsa Olsson’s (2011) thesis Den metakognitiva läraren – en pilotstudie om lärarens förhållningssätt och tillämpning av metakognitiva aspekter i undervisning studied teachers’ attitudes toward metacognition. The study used a survey that asked teachers to assess themselves with respect to four areas:

A. How much the teacher reflects upon metacognition in his/her planning of instruction.
B. The teacher’s attitude towards the term metacognition.
C. What the teacher does to promote metacognitive development.
D. What importance the teacher gives metacognition during assessment and grading.

The respondents came from one school in a suburb of Göteborg, Sweden. Sixteen teachers with different teaching experience, gender, and subject matter responded. All taught to students between ages 13-16 years (Swedish högstadiet).

Responses were analyzed quantitatively on a ten-point scale. Analysis was done to compare teachers with respect to experience, gender, and subject matter. An average point total was given to all subgroups. Results showed the following:

1. Although teachers (across all subgroups) put a relatively high amount of importance on metacognitive factors during assessment and the giving of grades, they did not report as much activity in teaching and training their students metacognitive understanding.
2. Teachers (across all subgroups) who understand how to plan for metacognitive development have a better understanding for how to nurture that development in their practice.
3. Female teachers had higher points across all four areas than male teachers.
4. Experienced teachers had higher points across categories A and B, whereas newly educated teachers had higher points across categories C and D.
5. Math and science teachers had lower points than Swedish and social studies teachers across all four categories and lower points than language teachers across categories A and C. Math and science teachers had higher points than
language teachers in category D and math and science teachers had the same amount of points as language teachers in category B.

In Olsson’s conclusion she discusses her results generally:

Sammanfattningsvis kan man efter denna undersökning konstatera att det finns ett behov av att fördjupa och utveckla lärarens förståelse för och användning av metakognitiva aspekter i undervisningen. Den metakognitiva aspekt där lärarna genomgående uppnår lägst medelvärde är den pedagogiska tillämpningen av metakognition, dvs. förmågan att träna sina elevers metakognitiva förståelse (kategori C). Detta innebär att de elever som inte är självlärda eller får del av denna kunskap på annat håll går miste om en viktig aspekt som bevisligen har goda effekter på lärande. Däremot visar undersökningen att de flesta lärare är relativt positiva till att ta hänsyn till metakognitiva aspekter vid betyg och bedömning (kategori D), något som verkar motsägelsefullt i förhållande till de låga poängen för kategori C (p. 25).

Here she states that there is a need to develop teachers’ understanding of metacognition and how to develop it in our students. In addition it is alarming to Olsson that although teachers need to develop a practical understanding of how to develop metacognition in our students we put weight on their metacognitive capabilities during assessment and grading, something that sends a contradictory message.

When discussing her results with regard to teachers’ subject matter Olsson states:

Slutligen visar resultaten att svensk- och SO-lärare uppnår högst medelvärde i alla kategorier utom i kategori C, där språklärarna får högst resultat. Vidare får matte- och NO-lärarna lägst medelvärde i alla kategorier utom kategori D, där språklärarna får lägst. Språklärare tenderar på så vis att göra en bättre pedagogisk tillämpning av metakognition än övriga lärare. Detta resultat kan delvis återspeglas i formuleringarna i kursmålen för de olika ämnena, där metakognitiva mål endast går att finna i kursmålen för svenska och moderna språk (p. 26).

Olsson says that teachers of Swedish and social studies receive highest points on the average across all four categories whereas teachers of math and science receive lowest points on the average across all categories except category D. She explains that this may be due to metacognitive learning goals not being present in math and science course plans.

Although Olsson’s rationale for the lower average point totals of math and science teachers may be true, the course plans and learning goals have been re-written since her thesis was written. It is therefore important to explore whether mathematics teachers have received the training necessary to help their students achieve these new learning goals. More importantly it is the impetus for the second question guiding this research: Do mathematics teachers feel a need to increase their level of expertise when developing their students’ metacognitive abilities?

If mathematics teachers themselves report that they feel less than adequately prepared to meet their students’ metacognitive needs and the research shows that increased metacognitive development is related to improved performance within mathematics, then we ought to explore ways of helping teachers who themselves have reported a need for improvement.
3.3 Teachers' intuitive declarative knowledge of metacognition

Anat Zohar’s (1999) study *Teachers’ metacognitive knowledge and the instruction of higher order thinking* examined “teachers’ intuitive knowledge regarding metacognition of thinking skills” (p. 418). Zohar defined metacognitive declarative knowledge as “an explicit awareness (that may described in words) of one’s reasoning patterns as well as the ability to think of (and talk about) reasoning patterns as distinct entities that may be related to specific tasks” (p. 416). Zohar wanted to see if teachers’ intuitive declarative metacognitive knowledge of thinking skills were sufficient to teach higher order thinking.

Zohar performed a qualitative study of teachers during in-service courses designed to prepare them for the implementation of Thinking in Science (TSC) lessons where “instruction of higher order thinking is integrated into the science curriculum rather than taught as a separate subject” (p. 416). Zohar analyzed data collected during the teaching courses and found “that teachers’ intuitive declarative metacognitive knowledge of thinking skills was found to be unsatisfactory for the purpose of teaching higher order thinking in science classrooms. A general practical implication from this finding is that courses which prepare teachers for instruction of higher order thinking should address extensively the issue of declarative metacognitive knowledge of thinking skills.” (p. 426)

Zohar’s conclusions reiterate the need for improved understanding of metacognition and its processes in teachers. Zohar’s study was specific to science instruction, but if the new Swedish learning goals also demand that students display these higher order thinking skills, than teachers in all subject areas must be prepared to meet the challenge. Zohar’s study was conducted in Israel, but her findings seem to reflect Olsson’s (2011 see above) conclusions that teachers of science and mathematics in a school in a suburb of Göteborg, Sweden do not incorporate the explicit teaching of metacognition in their practice. If similar results have been found in Israel and in a specific Swedish secondary school, can it be the case elsewhere?

4 Method

4.1 Qualitative research interview

The current research was performed using partially structured qualitative email interviews. Ryen (2004) describes qualitative research’s preference for:

1. Qualitative data in the form of words and pictures, not numbers.
2. Natural data through observations and unstructured interviews
3. Understanding before action, but from the perspective of the interviewer and interview subject.
4. Inductive hypothesis generating rather that hypothesis testing research (p. 16)

Kvale & Brinkman (2009) state that research interview’s goal is to produce knowledge that is constructed during the interaction between interviewer and the person being interviewed. One form of research interview is the partially structured
interview of one’s experienced world. In this type of interview the goal is to obtain descriptions of the interview subject’s lived experience in order to discern the significance of the described phenomena (Kvale & Brinkman, 2009). Proponents of quantitative research methodology may raise questions about qualitative methodologies subjectivity and it’s production of data that is unreliable. Kvale & Brinkman (2009) point to the importance of the interviewer’s ability to listen and ask follow-up questions and state that qualitative interviews have gained acceptance as a scientific method for many disciplines including pedagogy (p. 28).

It is the intention of this research to understand the experiences of secondary school mathematics teachers and how they have prepared for and work toward the development of their students’ metacognitive abilities as set forth in the new Swedish learning goals for mathematics courses. The interviews were partially structured as the initial questions were prepared, but follow-up questions were posed without a predetermined design. The research does not attempt to test a hypothesis, but rather to formulate conclusions for further study and consideration.

4.2 Email interview

The Internet revolution has provided researchers a new way of collecting data (Ryen, 2004). Some advantages to conducting email interviews are: it saves both time and money, it is not essential to schedule set interview times with interview subjects, one can conduct several interviews simultaneously, contents can include attached files, interview subjects write the research data themselves (no need for time consuming interview transcribing), and the interviewer has time to reflect over answers (Ryen, 2004, p. 197). Due to time constraints, email interviews were considered convenient. Interviews took place between a change of semester and the winter sports vacation in Sweden: a time-consuming period for many teachers. Additionally, due to the fact that the researcher is not a native speaker of Swedish and interviews were conducted entirely in Swedish, much time and misunderstanding could be saved if spoken interviews were avoided. This language difficulty would have been compounded if the interviewer did not have sufficient time to consider the nuances of responses before responding with properly phrased follow-up questions.

Email interviews are not free of disadvantages. Some of these include: considering who has access to the information obtained, variation of one’s ability to express themselves in writing (particularly online), creating trust and accuracy in the communication processes nuances without the benefit of body language (Ryen, 2004 and Kvale & Brinkman, 2009). It was assumed that all interview subjects were capable and experienced users of email because all interviews were conducted through their work-related email accounts. Email has become a primary means of obtaining information and communicating between entities (teacher-teacher, teacher-administrator, etc) at the school represented in the study. Although body language could not be interpreted, the trust and accuracy of responses was assumed due to interview subjects’ familiarity with the researcher as a previous and/or current colleague.

4.3 Trustworthiness/trovärdighet, reliability/tillförlitlighet, and generalizability

Denscombe (2004) defines trustworthiness (trovärdighet in Swedish) as the
researchers ability to provide enough information within the research report so that the reader can determine whether the research has been conducted with a high degree of precision. This requires that the report provides information on data collection and analysis and motivates chosen methodologies.

For Denscombe (2004), validity is an important aspect of a research’s precision. Face validity was improved by controlling the interview questions with the research mentor (Denscombe, 2004, p. 127). The data’s level of detail was high due to already transcribed responses. Accurate analysis of the data may be compromised due to the researcher having English as a first language and interviews being conducted in Swedish. It is difficult to ensure that interpretations of the nuances of responses were accurate. It is also difficult to ensure that responses came from the specific interview subjects, however responses originated from work email accounts protected by password and login information assumed to be private to the interview subject.

Another factor important to a research’s precision is its level of reliability (tillförlitlighet in Swedish). Denscombe (2004, p. 125) describes reliable methodologies as those that can produce similar results in different environments, by different researchers, with the same subjects during a different time, and with different groups of similar participants during the same time. Questions were very open and non-leading (see attachment in section 9.1 “Email interview sent to prospective subjects”). Interview subjects were given the freedom to answer the questions in their own way. This reduced the amount of bias that would be generated from encouraging them to answer in the way the researcher wanted, but detracted from the ability to assure that the questions posed produced data that can be reproduced. Repeating the same study with the same participants would not serve the purposes of this study because the teachers would become more informed on metacognition, what it means, and how they can contribute to their students’ development during repeated “trials”. In the long run, it is this researcher’s desire that more teachers become more informed, but the purpose of this study is to uncover if, at this moment, these participants have the understanding and knowledge of what is necessary to develop metacognition in their students. In addition, the researcher is/has been a colleague of some of the interview subjects. Interviews conducted by another researcher at another time cannot be assumed to yield similar results.

The use of qualitative interviews speaks to the researcher’s desire to try and understand these teachers’ experienced worlds: how they view metacognition and how they work in their daily practice to promote metacognitive development in their students. No attempts to generalize these teachers’ experiences to a larger population have been made, however it could be argued that these teachers responses are representative of mathematics instruction at the school in which they teach. To generalize the findings to other populations would require costly classroom observations with a randomly chosen representative group of teachers. Perhaps this study can provide the impetus for such a study.

4.4 Ethical concerns

Swedish Vetenskapsrådet (2009) describes eight different rules for ethical research that encompass four primary areas of concern: Informationskravet (information requirement), Samtyckeskravet (voluntary participation requirement),
Konfidentialitetskravet (confidentiality requirement), and Nyttjandeckravet (requirements for how participant information may be used). Below I discuss the measures taken during the interview process to ensure these rules were followed.

Informationskravet:
Regel 1 Forskaren skall informera uppgiftslämnare och undersökningsdeltagare om deras uppgift i projektet och vilka villkor som gäller för deras deltagande. De skall därvid upplysas om att deltagandet är frivilligt och om att de har rätt att avbryta sin medverkan. Informationen skall omfatta alla de inslag i den aktuella undersöknugen som rimligen kan tänkas påverka deras villighet att delta (Vetenskapsrådet, 2009, p.7).

Interview participants were informed that their participation required only their responses to the email interview and follow-up questions. Participants were also given the contact information of the researcher’s mentor for this study.

Samtyckeskravet:
Regel 2 Forskaren skall inhämta uppgiftslämnares och undersökningsdeltagares samtycke. I vissa fall bör samtycke dessutom inhämtas från förälder/vårdnadshavare (t.ex. om de undersökta är under 15 år och undersökningen är av etiskt känslig karaktär).

Regel 3 De som medverkar i en undersökning skall ha rätt att självständigt bestämma om, hur länge och på vilka villkor de skall delta. De skall kunna avbryta sin medverkan utan att detta medför negativa följer för dem.


Interview participants were informed on their right to stop the interview process whenever they so chose. Interview candidates were reminded of their voluntary participation and given the choice not to participate when reminder emails were sent. Participants who chose not to participate were emailed directly and thanked for their non-participation response.

Konfidentialitetskravet:
Regel 5 All personal i forskningsprojekt som omfattar användning av etiskt känsliga uppgifter om enskilda, identifierbara personer bör underteckna en förbindelse om tystnadsplicht beträffande sådana uppgifter.


Interview subjects were informed that their participation would be held confidential during the reporting of the study. All interview responses were stored in a secure email account for which only the researcher has password and login information.

Nyttjandeckravet:
Although participants were not informed of this right at the start of the interview process, information obtained in connection to the study has not been used for any other purpose than the current research.

4.5 Selection of interview participants

Prospective interview subjects were selected because of convenience. Requests for participation were sent to sixteen mathematics teachers and five responded. They were relatively easy to contact due to my previous/current affiliation with the school as a substitute/intern and include teachers from a secondary school on Sweden’s west coast. All teach/have taught mathematics at Swedish secondary school (gymnasieskolan).

Of the eleven teachers who did not participate, four indicated that they did not want to participate, one indicated that they did not have the time or energy, one was working as an intern during teacher education and was therefore not credentialed, one did not teach any courses from the new school reform (Gy11) and was retiring at the end of the school year, one indicated the possibility of participation only if absolutely necessary to the research (they did not respond to a final reminder), and three teachers did not respond after three reminders.

No attempts have been made to have a random or representative sample of participants because it was each individual teacher’s experiences that were of interest.

4.6 Theoretical background for interview questions

Göran Svanelid at Stockholm University has studied the new school guidelines and learning goals within Swedish primary schools (Lgr 11) and what he terms “The big 5” abilities/capacities have emerged (online source). These are: analysis, communication, handling information, understanding concepts and the relationship between them, and metacognition. Svanelid has made “The big 5” more concrete by interpreting each capacity and how it is expressed within Lgr 11. Svanelid has identified the following terms within Lgr 11 that relate to the metacognitive capacity: tolka (translate); värdera (evaluate); ha omdömen om (judge); reflektera (reflect); lösa problem med anpassning till en viss situation, syfte eller sammanhang (solve problems with adaptation to a specific situation, purpose, or context); avgöra rimligheten (determine reasonableness); välja mellan olika strategier (choose between different strategies); pröva och ompröva (test and reconsider).

Svanelid’s metacognitive terms from Lgr 11 can be easily tied to the definition of metacognition and the teaching strategies that support it. Wilson’s (1997, p5) definition of metacognition as “the awareness individuals have of their own thinking and their ability to evaluate and regulate their own thinking” includes one of Svanelid’s terms: evaluate. Students should be engaged in the evaluation both of their
work and of their learning process. Ramdass & Zimmerman (2008) advocate the use of accuracy training techniques so that students may evaluate their learning more accurately.

Svanelid includes the term “judge.” When students are engaged in making judgements about their own learning they improve their self-regulation. Zimmerman (2002) states that improved self-regulatory skills improve academic performance. Improving these skills can be supported by the use of metacognitive questioning strategies (Sjöström, 1998), summarizing (Bondy, 1984), and techniques for guiding self-efficacy (Chen, 2003) thereby improving the effective allocation of study time (Pashler et al., 2007).

Svanelid’s inclusion of “reflection” can be related to Flavell’s (1971) initial definition of metacognition: thinking about one’s own thinking. To achieve this one must reflect upon oneself. Dysthe, Hertzberg & Løkensgard (2002) included the aspect of awareness to the definition of metacognition. To be aware of one’s learning requires that one reflect upon it. In addition to questioning strategies recommended by Dimenäs (1995) and Sjöström (1998) and summarizing recommended by Bondy (1984), debriefing techniques (Leat & Lin, 2003) and journal writing (Bondy, 1984 and Theobald, 2006) can be used to support student reflections.

Svanelid also discusses problem solving and its relationship to a metacognitive ability. Schoenfeld (1985) states that students adept at problem solving engage in an internal dialogue where they constantly evaluate the evolution of their solutions and not just the solutions themselves. To support students’ development in this internal dialogue (which also involves what Svanelid calls “test and reconsider”), Bondy (1984) suggests the use of teacher think-alouds.

Choosing between different strategies is mentioned by Svanelid and can also be supported by think-alouds. Baker and Brown (1984) included self-regulatory mechanisms for checking the outcome, planning, monitoring effectiveness, testing, revising, and evaluating strategies in their definition of metacognition. This definition also supports Svanelid’s inclusion of strategy choosing. Baker and Brown’s (1984) definition of metacognition including “checking the outcome” also supports what Svanelid calls determining reasonableness.

The interview revolves around the learning goal (kunskapskrav) that requires Swedish teachers to develop and assess their students’ capacities in relationship to metacognition:

Eleven kan utvärdera med nyanserade omdömen resultatets rimlighet samt valda modeller, strategier, metoder och alternativ till dem
(Skolverket, 2011).

This learning goal was chosen due to its (1) reiteration of the terms that Svanelid recognizes as having a metacognitive purpose within GY11: omdömen (judgement), rimlighet (reasonableness), strategier (strategies), and utvärdera (evaluate) and (2) because the same learning goal is stated in every mathematics course within Swedish secondary schools and therefore is relevant to all who are/going to be teaching courses from GY11.
Teachers were asked to report their professional background information including gender, number of years teaching mathematics, other subjects they teach, educational background, current form of school in which they are currently employed, and math courses they currently teach. Although no attempts were made to make the study generalizable to a larger population, the background information was seen necessary in order to describe the breadth of qualities present in the interview respondents. They were then asked to describe how they work toward this learning goal, both the opportunities and difficulties they perceive and encounter when working toward this goal, and anything they thought could make their work towards this goal easier.

4.7 Self critique

The use of self-reporting as primary means of assembling data for analysis can be questioned. Classroom observations of teachers over a longer period of time could be a better means of assessing what teachers actually do to promote metacognitive development (more costly: time/money) because teachers may not be cognizant of how their strategies contribute to the development of metacognition in their students. This researcher did not have the time or money to conduct a study of that type.

Translation of Swedish research and Swedish interview responses as well as formulating interview questions in Swedish when the thought process of the interviewer is in English can lead to misinterpretation. However, the research mentor provided assistance in the formulation of interview questions and the accurate interpretation of interview responses.

Email interviews were not only used to save time and provide simultaneous access to multiple participants, but also to assist in ensuring the accuracy of the data collected. The use of email interviews cannot provide the same type of information that face-to-face interviews can, but this was weighed against the disadvantages the researcher would have had in face to face interviews due to language issues. Email interviews provided the researcher time to think about the complexity of the responses and adequately formulate follow up questions in Swedish.

One cannot assume that question 2 has been completely answered. It is difficult to interpret how someone feels from written responses, but written indicators of emotional states were used (exclamation points, capital letters, repetition, etc).

5 Analysis

Because the goal of the research interview was to obtain descriptions of the interview subject’s lived experience in order to discern the significance of the described phenomena (Kvale & Brinkman, 2009), the interviews were analyzed/interpreted hermeneutically. A hermeneutic analysis/interpretation both highlights the interview process and the interaction between the interviewer and respondent as well as provides a model for how the interview is to be analyzed/interpreted (Kvale & Brinkman, 2009). Kvale & Brinkman (2009) describe the circular nature of hermeneutic analysis where the whole affects the interpretation of its parts, which in turn affects the interpretation of the whole. The interviews were read first in their entirety to get an understanding for the respondent and how they perceived both the
questions and their own reality in relationship to them and their teaching practice. Interviews were read again looking for specific phrases that could indicate either the use of a specific teaching strategy for supporting metacognition or the respondents’ emotions relating to their expertise of metacognition and/or their ability to answer the questions. When necessary, follow up questions were sent as quickly as the initial analysis/interpretation could be done so that respondents may clarify their responses. Because interviews were conducted entirely in Swedish, the initial interpretation required some time thereby delaying some follow-up requests for clarification. Additionally, the presentation of the interviews seeks to both highlight the interaction between interviewer and respondent as well as clarify the significance of the Swedish text for the reader.

Response lengths varied from one half page with no attached documents to five pages with twelve attached documents. Four of the five participants were men. All participants are credentialed mathematics teachers in Swedish secondary schools (gymnasiet) and adult education (KomVux). One of the participants is also a credentialed elementary school teacher in Sweden in mathematics, physics, chemistry, and biology. Three of the five participants are currently teaching courses from GY11. The participants’ years of teaching experience range from two years to 34 years. Secondary school teachers in Sweden are required to have credentials in more than one subject. The participants also teach physics (2 participants), computer science (1), “teknik” (1), and natural science (1).

Because the research poses two separate questions they will be presented separately. First with regard to question 1: Does teacher practice match the current research on metacognitive mathematics instruction? And then with regard to question 2: Do mathematics teachers feel a need to increase their level of expertise when developing their students’ metacognitive abilities?

5.1 Does teacher practice match the current research on metacognitive mathematics instruction?

In order to answer research question 1: “Does teacher practice match the current research on metacognitive mathematics instruction?” interview responses were analyzed/interpreted in order to connect them to the nine different teaching strategies for developing students’ metacognitive abilities (see “teaching metacognition” earlier):

1. Questioning and questioning strategies
2. Group discussions
3. Think-alouds
4. Summarizing
5. Debriefing
6. Journal writing
7. Guiding self-efficacy
8. Accuracy training
9. Efficient allocation of study time

A brief visual representation of the group (an “X” denotes a strategy reported by that teacher):
Table 1: Teachers and the metacognitive teaching strategies they report

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Questioning and questioning</td>
<td>A</td>
</tr>
<tr>
<td>2. Group discussions</td>
<td>B</td>
</tr>
<tr>
<td>3. Think-alouds</td>
<td>C</td>
</tr>
<tr>
<td>4. Summarizing</td>
<td>D</td>
</tr>
<tr>
<td>5. Debriefing</td>
<td>E</td>
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<tr>
<td>6. Journal writing</td>
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<tr>
<td>7. Guiding self-efficacy</td>
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<tr>
<td>8. Accuracy training</td>
<td></td>
</tr>
<tr>
<td>9. Efficient allocation of study</td>
<td></td>
</tr>
</tbody>
</table>

An analysis of each teacher’s responses:

**Teacher A** does not currently teach any courses from GY11 and at this time works exclusively within adult education (KomVux). He described in detail important components in defining an answer’s reasonableness as well as different strategies and methods. There were a number of statements that indicate his awareness of teaching strategies that encourage metacognitive growth. The first of these refers to improving self-efficacy in students who have not succeeded earlier with mathematics:

"För deras del handlar det om att deras matematiska självförtroende skall stärkas. Det gör jag genom att välja enkla uppgifter i början för att sedan gå mot större kompexitet."

"Samtidigt är det så att min uppgift är att få eleverna att veta när de förstått och för att förstå en strategi behöver man ha språket."

His response indicates his awareness of his students’ low self-efficacy beliefs, but not necessarily the specific teaching strategies he uses to improve these beliefs. When asked to describe the strategies and methods he uses to develop his students’ ability to "utvärdera med nyanserade omdömen,” he answers:

"Det jag egentligen säger är att jag försöker fånga upp eleverna där de befinner sig i sin kunskapsutveckling. Jag försöker att inte skjuta över målet utan anpassa undervisningen till elevernas förkunskaper, och ställa mina krav lite högre så att de utvecklas matematiskt."

His approach is very much in line with what Vygotsky calls "zone of proximal development” or ZPD. He hopes to meet the students where they are now in their development to sequentially lead them to deeper understandings.

He goes on to say:

"Så här brukar jag göra. Låt eleverna fundera på en uppgift från det centrala innehållet. Därefter fänger jag upp hur de har resonerat, där de får förklara sin metod, vilket ger träning i resonemangsförmåga. Vid flera sätt att lösa en uppgift tittar vi på vilken metod/strategi som verkar enklast.”
Here he is very concrete in what he does, but a little vague in how he does it. He states that "fångar jag upp hur de har resonerat, där de får förklara sin metod” meaning that he captures how they have reasoned when they explain their method. He has his students reflect upon their methods but it is unclear whether they work individually, in groups, if he captures their reasoning based upon discussion or analysis of written work. He then describes how he effectively uses a scaffolding technique to stretch the students’ capabilities with regard to specific subject matter content.

He describes also that he uses several different types of representations to illustrate mathematical content:

"På A-kursen tränar jag dem att få fram mönser ur talserier och även ur geometriska mönser. Både färdigritade och sådana där de själva får bygga upp mönster med tändstickor eller annat handson material."

"För att förstå en metod/strategi försöker jag alltid att skapa bilder, mentala bilder som skapas med språket vilket inkluderar kroppsspråket, eller bilder ritade på tavlan. Det är helt naturligt på geometriavsnitt men jag försöker göra det så ofta jag kan…När elever arbetar kan det ibland vara en fördel att använda hands on material i stället för att bara rita figurer. Jag gör det på en del avsnitt och framförallt när jag vill att eleverna skall få fram mönster dvs finna en formel eller när vi arbetar med funktioner."

He highlights the use of different representations: mental pictures created by language and body language, pictures drawn on the white board, and hands-on material. Although not one of the nine methodologies for the basis of analysis here, Kahveci & Imamoglu (2007) suggest that this practice does yield positive results in the mathematics classroom.

Teacher A attached a number of documents that illustrate other ways he helps his students. One of these is giving the students a detailed planning for the entire course. This information can help the students know what the learning goals are for each meeting. Bondy (1984) calls this identifying learning goals and it can be seen as a form of written think-aloud. Teacher A describes another method he uses to help students understand learning goals:

"En annan metod jag använder mig av för att få eleverna att utveckla metoder/strategier och modeller är att visa på målet med momentet Se bifogade statistikmål för kurs A eller att börja med en sammanfattning."

Here he describes how, by "starting at the end” with summaries or explicitly stating learning goals, he can illustrate for students what they are expected to know/be able to do at the conclusion of a particular mathematical content area. He concludes by saying:

"Med ovanstående målformulering hoppas jag att eleverna ser progressionen i vad de kan och vad jag vill att de skall kunna.”

Again he has illustrated (he hopes that students see the progression in what they already are able to do and what he wants them to learn) a technique of describing learning goals for students that can help them become more accurate in their self-efficacy beliefs.
Another attached document included the teacher’s planning and handouts for teaching applications of quadratic functions. His handouts included detailed solutions for the students so that they may check their accuracy and method. Although this attachment was not specifically described as a test or a quiz and it is not known whether the results of student work were turned in or reflected upon formally, it can be interpreted as similar to what Pashler et al. (2007) suggest: that teachers use assessments (tests and quizzes) to identify for students what they still need to learn which is a form of accuracy training that can lead to improved self-efficacy beliefs (Ramdass & Zimmerman, 2008).

Teacher A also attached a document for use during a group task. He describes the use of the group tasks in his classroom:

"Ett tredje sätt att arbeta är att låta eleverna lösa gruppuppgifter. Jag har ett antal så kallade "Gemensam problemlösning" där varje elev i gruppen får 1 st information var. För att besvara frågorna krävs alla informationerna och därmed måste alla i gruppen förstå och alla behöver hjälpas åt för att lösa uppgiften”

Students are given one piece of information each, but all information is required to solve the problem. In order to solve the problem it requires that all students understand their information piece and can help one another create a strategy or model that incorporates everyone’s information. Without having witnessed groups working this way, I can only assume that much of the conversation revolves around the proposing and constant reforming of possible methods that students can use to solve the problem. This would lead to deep and meaningful exchanges between students about the advantages and disadvantages of various proposed methods and strategies. This is seen as a good example of a group discussion that can lead to metacognitive development (Paris, 1984).

**Teacher B**’s initial reaction to my questions were interesting:

"Hmm... Jag hittar inte denna exakta formulering i ämnesplanen för Matematik...”

It should be noted here that my reference to the learning goal within mathematics was formulated from the criteria for the highest possible grade. The goal is formulated in varying degrees of complexity to differentiate grading levels. I chose the highest level because I assume that we as teachers should strive to encourage students to achieve the highest of all possible levels. Of course to achieve the highest level we must meet students where they are in their development.

Teacher B continues his comments on how he works toward the goal with group discussions (whole class):

"Väljer ut, genomför och diskuterar olika problem och lösningar på tavlan i helklass. Uppmanar enskilda elever att fundera över detta, i samband med individuellt hjälp.”

Teacher B not only comments on the use of group discussions, but also that the subject is discussed when helping students individually. He was asked then to describe more thoroughly how these discussions might look like in his classroom. He discusses briefly two different examples of problems involving quadratic functions
and then says:

"Jag ritade då upp två grafiska illustrationer av problemet; diskuterade den matematiska modellens och lösningarnas giltighet; Vikten av att motivera / kommunicera sina antaganden och angreppssätt, m.m."

"Här finns mycket att diskutera, både egna (rimliga) antaganden och deras redovisning; och det faktum att den matematiska modellen måste användas med försiktighet -- grafen till andragradsfunktionen överensstämmer ju bara delvis med vattenstrålen. O.s.v."

He indicates that the particular example problems (see attached interviews) lend themselves well to utilizing group discussion to analyze reasonableness and judging the effectiveness of mathematical models. He also describes how he uses multiple representations of the problems "Jag ritade upp två grafiska illustrationer av problemet…" Very much like Teacher A, Teacher B uses a strategy that Kahveci & Imamoglu (2007) suggest that yields positive results in the mathematics classroom.

Teacher B also attached both a test and answer key that he made that he uses to help students understand both how they are assessed and reflect upon their own solutions with respect to GY11’s grading criteria:

"Bifogar ett delvis "hemmagjort" prov, utefter de nya bedömningskriterierna. Jag har diskuterat dessa MYCKET med eleverna: de är väl medvetna om de sju förmågorna, och jag brukar använda /exemplifiera dem vid genomgångar. Jag ger också eleverna både facit och mina egna bedömningsanvisningar till varje prov så att de kan gå igenom och jämföra."

He states that he discusses the grading criteria "MYCKET” (A LOT) with his students, that they are aware of "de sju förmågorna” (the seven abilities/capabilities), and that he uses and illustrates them during lectures. Here the teacher has referred to seven abilities/capabilities that the University of Stockholm has described as underlying the mathematics courses’ grading criteria. Three of which contain components and terms that relate to metacognition described by Göran Svanelid: problemlösningsförmåga ("värdera valda strategier, metoder och resultat"), modelleringsförmåga ("utvärdera en modells egenskaper och begränsningar"), and resonemangs förmåga ("bedöma matematiska resonemang").

He was then asked to elaborate on exactly what he means by “de kan gå igenom och jämföra” (they can go through solutions and compare):

Först och främst gör vi detta gemensamt, så att de förstår exakt hur jag har resonerat. Sedan uppmuntrar jag dem att själva, i efterhand, jämföra sina egna lösningar (och mina bedömmningar av dem) med "facit" och min poängsättning; och att FRÅGA vid oklarheter. Jag uppmuntrar dem, generellt, till att hela tiden fråga sig "vad kan bli bättre med mina lösningar?"

Again he talks of the use of whole class discussion but includes that he does this so that “de förstår exakt hur jag har resonerat” (they understand exactly how I have reasoned). This approach can be considered a form of a think-aloud (Bondy, 1984). This combined with his highlighting how often he refers to the learning goals and capabilities/abilities in the Swedish course plans also points to him modeling his metacognitive thoughts (Bondy, 1984). He also talks of how he encourages students to compare their solutions to those found in the answer key provided on their own
time. This can be seen as a form of accuracy training that leads to more accurate self-efficacy especially combined with the question he wants students to ask of themselves, “vad kan bli bättre med mina lösningar?” (Bandura, 2006). This question also displays the use of a self-questioning technique that promotes metacognitive development (Dimenäs, 1995).

Teacher C’s first comment in his responses revolved around the weight he (and presumably) other teacher’s give the learning goal:

“Målet du nämner är väl ett av dem som inte alltid kommer högst upp på listan och får mest tid. Ibland får man vara glad om eleverna överhuvudtaget löser problemen.”

In contrast to earlier learning goals and grading criteria in the Swedish course plans, students must now be able to show their ability to “utvärdera resultatets rimlighet samt valda modeller, strategier, metoder.” According to GY11 a student may not receive a passing grade if they cannot show this ability. It is the degree to which a student displays this ability that determines which passing grade he/she receives. Accepting the Teacher’s response (and what appears to be an assumption about what happens in mathematics classrooms in general) highlights Olsson’s (2011) findings that teachers reported grading based upon metacognition without instructing students’ development of it.

The teacher also reports that he does not have a specific strategy when working with this learning goal, but rather addresses it spontaneously when appropriate with a particular task (a comment that we will see again from other teachers):

"Jag har nog ingen egentlig strategi för att jobba med dessa mål, utan jag brukar ta upp dem då och då spontant när det passar med någon uppgift vi håller på med. Vissa uppgifter är ju ganska enkla att visa med olika lösningsmetoder, medan andra kanske är mer ensidiga."

He goes on to comment on how sometimes addressing the goal is easier when he is instructing in physics (he often has the same students in both subjects):

"Ofta har jag samma elever i matematik som jag också har i fysik, och dessa frågor dyker upp även där: Är svaret rimligt, kunde man gjort på något annat sätt? I fysik kan det ofta vara lättare att jobba med detta tycker jag, men jag tror att det "spiller över" på elevernas matteinlärning också.”

Here the question indicated by the teacher “Är svaret rimligt, kunde man gjort på något annat sätt?” causes the students to think about the contents of the learning goal, namely to judge an answer’s reasonableness and compare problem solving methods. It is difficult to make conclusions without having observed this teacher in the classroom during this type of conversation, but it would be interesting to know if he also scaffolds the students thinking by asking ”deep, explanatory” (see Hemberg, 1995 and Dimenäs, 1995 above in teaching metacognition) questions such as, ”Why or why not is the answer reasonable?” or ”How do you know the answer is reasonable or not?” or ”Under what circumstances are the different solving methods better/worse than the others?”

After considering the possibility of the teacher utilizing questions like those above a follow-up question was sent because I was more interested to know how the
discussion process looks in his classroom: “Kan du berätta hur det kan se ut när du tar upp sådana problem som passar bra? Det kanske kan hjälpa att upplysa om du kommer ihåg en specifik situation där det hände :)” His response:

"Tydliga exempel som jag kommer på nu är inom geometri där man kan få enklare lösningar om man väger sitt x på lämpligt sätt. Konkret exempel kan jag inte komma på just nu.

Igår skulle en elev derivera $(x^2 + 3)^2$ och valde att skriva ut parentserna var för sig och sedan derivera som en produkt, jag föreslog att man skulle kunna använda inre derivata istället. Det visade sig att eleven inte riktigt fattat inre derivata och därför valde produktvarianten. Men efter lite hjälp insåg han att inre derivata var smart."

Although the answer showed that the teacher assists students to come to the realization that different methods can yield the same result and that these different methods ought to be compared, it is unclear how he helped the student reach this understanding (in this case with regard to inner derivatives). For non-math teachers the specific example is not very interesting. What is particularly interesting is that the teacher chose to focus on the mathematical content that triggered the interaction between himself and the student rather than the interaction’s underlying structure and processes. This could suggest that the teacher either is more focused on the mathematical content rather than the processes that underlie the learning of it or it could be that the teacher was not really clear about how I wanted him to answer the question. In either case, the question was intentionally phrased as an open question in order to allow the teacher to answer the question in his way so that I may have a greater understanding of how he thinks about his practice.

The teacher also reflects upon another strategy that could help students’ development toward the goal:

"En sak jag kom att tänka på nu, men som jag aldrig provat, är att man skulle kunna dela in klassen i mindre grupper 4-5 elever i varje, ge dem en uppgift som de får jobba med och fundera över om det finns olika sätt att lösa den, sedan kan man tillsammans i hela klassen gå igenom några olika varianterna och diskutera dem, deras fördelar och nackdelar."

The teacher has reflected upon his own practice and found another methodology that enhances metacognition: group discussions. From his earlier comments, we can conclude that he already uses group discussions (whole class) but has realized that structuring these discussions with fewer students in each group could help the students more. This practice could encourage participation from more students as suggested by Kahveci & Imamoglu (2007).

**Teacher D** noticed that I had chosen the learning goal in its most complex form (see also Teacher B). He discussed how they work towards this goal in its less complex form (those criteria students must achieve to receive a passing course grade) first with regard to the analysis of an answers reasonableness:

"Eftersom kravet förekommer med olika komplexitet på lägre nivåer jobbar vi givetvis med att göra rimlighetsanalyser i någon mån. Detta sker ofta genom jämförelser med för eleverna redan känta objekt, eller händelser. På grund av att jag har den klass som studerar med dessa ämneskrav i matematik även i fysikämnet harnar mycket av rimlighetsanalysen i detta ämne istället – det är helt enkelt lättare att hitta bra exempel
Here he describes that they often discuss the reasonableness of answers with situations and objects that they already have understanding of ("redan kända objekt, eller händelser"). He also states that because he teaches the same students in physics that he has in mathematics, the work of analyzing the reasonableness of answers often happens more naturally within his physics classroom, much like Teacher C.

After the teacher was asked to describe in more detail how the process of discussing and analyzing the reasonableness of answers, he responded by describing an example of what could be the basis for this type of discussion and also commented on how these moments often come up spontaneously (again like Teacher C) rather than being planned for ahead of time:

"Rimlighetsanalyser med redan kända objekt" kan vara exempelvis vara att man jämför en hastighet med ungefär hur fort den snabbaste människan springer...Tyvärr kommer dessa moment mer spontant - de uppstår när eleverna själva gjort något fel, inte när det är planerat en sådan analys."

Much like Teacher C, this teacher has chosen to focus on the mathematical content in his answer that triggers such inquiry rather than the inquiry process. I also find it interesting that the teacher uses the word "tyvärr" (unfortunately). It may be assumed that the teacher would like to plan for these moments to come up more regularly. It may also denote a sense of frustration on behalf of the teacher over things that may interfere with his ability to teach toward this learning goal in the way that he would like or in the way that he feels is required.

The teacher also commented that they usually compare different problem solving methods:

"Under lektioner och på prov brukar vi jämföra olika lösningar på problem för att avgöra vilket som i det specifika fallet är den bästa lösningen. Under genomgång av sådana problem jämför vi och visar på skillnader."

The teacher was also asked to describe in more detail how this process might look in his classroom. His response:

"Det är lite lättare att planera att jämföra strategier, men jag har ingenting nedskrivet på denna kursen ännu, så rent generellt handlar det om att vi räknar exempelvis en uppgift (på provet senast)...Jämförelsearbetet blir då att man visar lösningarna och visar varför svaret blir samma och sen blir det det en diskussion om vilken metod som är enklast...Arbetsformen är typiskt en lärarledad diskussion vid tavlan, oftast då med störst inblandning från de mest intresserade eleverna (det kanske finns strategier för att alla ska få komma till tals, men det är svårt just nu.)"

The teacher again discusses that, although it is easier to plan for the comparison of different problem solving strategies (as compared to planning for the analysis of an answer’s reasonableness), he has yet to have planned for it. He says that, generally, they look at problems that have achieved the same answer through different strategies and discuss which strategy is simplest. The discussion is often teacher-led at the white board and often driven by those students who are most ”interested.” The teacher mentions that there may be other teaching strategies to draw in other less-interested students, but that ”it is difficult right now.” This teacher has described that
he is aware of other strategies, but there is some kind of obstruction to the implementation of such strategies.

**Teacher E** teaches mathematics to students in work preparatory classes (yrkesprogram). She expresses the difficulty she has in motivating students to see mathematics’ importance and stimulating their self-esteem within mathematics:

"Många elever är inte bara svagpresterande utan saknar en grundläggande bas att bygga på: de kan helt enkelt inte räkna och är övertygande om att det är omöjligt för dem att lära sig matematik."

She also describes that, instead of limiting the course goals for these challenged students, she wants them to above all else develop belief in their own abilities, to overcome challenges, and learn to think and analyze, reason, and act:

"Genom min undervisning vill jag framförallt skapa tilltro till sin egen förmåga hos mina elever. Detta gör jag inte genom att erbjuda eleverna begränsad, i förhållande till kursplanens mål, undervisning, utan genom att få dem att klara utmaningar. Jag vill lära dem tänka och analysera, resonera och agera."

When asked to clarify what strategies she employs to work towards improving the students’ beliefs in themselves, she answers:

"Jag har delat en (våldigt svagpresterande) klass i två mindre grupper. Det blir mer tid för varje elev och inga störningar. Eleverna känner sig trygga och vågar mer i mindre klasser."

Here the teacher indicates her belief that dividing the class into smaller groups will provide students the opportunity to work in an environment with less disturbances and that the students will feel more secure and thereby increase their participation.

**5.2 Do mathematics teachers feel a need to increase their level of expertise when developing their students’ metacognitive abilities?**

In order to discern how the teachers felt about their abilities in helping their students meet the learning goal they were asked to answer the following questions: "Hur fungerar det för dig?" (How is it working for you?), "Vilka möjligheter/svårigheter upplever du?" (What opportunities/difficulties do you experience?), and "Vad skulle underlätta för dig för att hjälpa eleverna att nå kravet?" (What would simplify your work in helping students meet the goal?). Again I discuss the responses received to these questions for each teacher.

**Teacher A** names three factors that make his work more difficult. The first factor is time:

"När elever arbetar kan det ibland vara en fördel att använda hands on material i stället för att bara rita figurer. Jag gör det på en del avsnitt och framförallt när jag vill att eleverna skall få fram mönster dvs finna en formel eller när vi arbetar med funktioner. Oftast räcker inte tiden till för det. Lite längre kurstid skulle sitta fint."

"Lite längre kurstider skulle vara bra för att variera matematiken lite mer och därmed kunna använda laborativt materiel och låta eleverna samarbeta på ett mer styrt sätt."
In both of the above quotes the teacher discusses how more time would allow him to both work with hands-on materials and to allow students to cooperate in a more structured way.

The second factor Teacher A discusses is his students’ level of prior knowledge:

"Eftersom många vuxna kommer tillbaka till studier efter olika långa uppehåll skulle det underlätta mycket om de blivande kursdeltagarna hade gjort en test på sina förkunskaper och utifrån den repeterat kursen innan. Om så vore möjligt skulle det minska avhoppen och fler vuxna skulle inte gripas av panik att de inte förstår matematiken framförallt på B-E-kurserna.”


"Med dålig förförståelse tar tid att utveckla en förmåga så att den når upp till A-nivå.”

In his first reference to the difficulties posed with students who have insufficient prior knowledge, the teacher also identifies a possibility for improvement of students’ placement within mathematics courses: a pre-test. He states that placing students in courses that required them to repeat content for which they have not shown understanding would reduce the amount of students who become very stressed over the expectations placed upon them in higher level math courses and therefore drop out of these courses. This type of procedure can be directly linked to the strategy Pashler et al. (2007) suggest for supporting students metacognition. This type of test can not only serve to place students correctly, but also inform the students on what they do and do not know so that they may more effectively manage their time for study.

Teacher A’s second reference to students’ previous knowledge comes under discussion of the learning goal referred to in the study. Because I chose the goal in its most advanced and complex form, the teacher discusses how difficult is for students to achieve the goal completely. His reasoning for this difficulty lies in their previous knowledge and the goal of obtaining a grade that allows them to apply for higher education that many of his students have. It can be assumed then that the students do not have goals of achieving deeper understanding, but that they are more interested in the grade (or product) that comes at the end of the course. It should again be noted that Teacher A does not currently teach any courses outlined in GY11. When KomVux adopts similar course plans, a whole new set of demands will be placed on students in order for them to achieve the grades needed to seek higher education.

His final mention of prior knowledge relates to how difficult it is for students to achieve the highest possible grade and connects this back to the first factor discussed: time.

The last factor that Teacher A mentions is group size:

"Efterhand som eleverna lär sig mer matematik när de kommer högre upp i kurserna ser jag att en del kan sätta tidigare kunskaper i relation till detta mål Det kan också gynnas av att det inte är så stora grupper på C-kursen i matematik.”

He not only mentions that smaller group size contributes to working towards the
learning goal, but he also hints at his perception that working towards the learning goal, requires the student to have an understanding of more mathematical content.

**Teacher B** identifies two factors that work against him, the first of these being time:

"Ett HINDER för att arbeta mer med detta är att vi sällan har tid att diskutera vidare om "gamla" uppgifter innan vi måste gå vidare i det centrala innehållet. Detta är delvis ett lokalt problem med schemaläggning, men har också att göra med att det centrala innehållet är digert i förhållande till de 100 poängen (vilka blir c:a 75 timmar i verkligheten)."

He discusses the limitations of the school schedule in which he works where students are not, according to him, given the time necessary to learn all of the content required in a course. The time limitation affects their ability to discuss "old" problems, before they are required to move on to the next content area. This teacher seems to organize his courses from the content to be learned and not from the abilities/capabilities in GY11. When responding to the question, "Vad skulle underlättta för dig…?" he repeats his previous statement quite clearly:

Mer tid / mindre stoff.”

The second factor that negatively affects his work is the availability of published national test problems:

"Möjligheten att arbeta med nationella prov. Just nu finns inga (ej sekretessbelagda, "gamla") prov att tillgå. Men även om det fanns, hade jag helst velat gå tillbaka och diskutera uppgifter på det nationella prov som eleverna faktiskt har skrivit (innan jul). Men det får jag ju inte! Just det där att "dissekera" uppgifter som eleverna har lagt mycket kraft på vid prov tror jag är ett perfekt sätt att få dem mer kvalitetsmedvetna och öppna för att ta in nya alternativa tänkesätt.”

In Sweden, previous national tests are published online after a period of time. Teachers and students have access to these older tests and can use them not only to prepare from, but also to gain information on how they are to interpret the learning goals in the current course plans. At this time, because GY 11 is in its first year of implementation, there are no such old national tests to use. He discusses how not only having access to older tests would help his work, but also that he would like to be able to use the tests that his students had just completed as a basis for deeper, meaningful discussion. Unfortunately he is not allowed to do this. His reflects on this with what must be interpreted as irritation or frustration: “Men det får jag ju inte!”

**Teacher C** speaks of one aspect that would simplify his work, time:

"Det som skulle underlättat vore såklart om man hade mer tid på kursen så att man inte kände sig stressad över att uppnå de mer grundläggande och "viktiga" målen.”

His response not only identifies the difficulty of too little time, but also points to the stress he experiences in his work and how he prioritizes certain learning goals over others ("så att man inte kände sig stressed over att uppnå de mer grundläggande och ‘vikti..."
That means that a student with extremely high levels of proficiency in all grading criteria, but not shown minimal proficiency in just one of them may not receive a passing grade. If teachers’ prioritize certain learning goals, then we thereby limit our students’ possibility of achieving higher levels.

**Teacher D** identified two primary factors that make the job difficult. The first factor he names is time:

"Det största problemet för att uppnå kunskapskravet är att det inte finns tid till att träna på saker som ger en djupare förståelse för matematiken – det centrala innehållet tar så pass stor tid i anspråk redan att på en 86-timmarskurs blir det väldigt lite tid över till reflection och gruppdiskussioner kring öppna problem."

GY11, in addition to modifying the way in which different abilities are weighted in grading, also has defined more specifically the content to be taught in each course (centrala innehållet). The teacher expresses a difficulty in managing the time given for each course in such a way that deep understanding can be achieved. This seems to point to indicate that it is the courses content (innehåll) that guides his teaching and not the abilities/capabilities (Stockholm Universitet’s sju förmågor). It appears that the teacher feels that deeper levels of thought can only come after specific content knowledge is learned as opposed to weaving the two together.

The second factor the teacher names is the students prior knowledge which he also states effects the time he can allot to deeper understanding:

"Ytterligare en faktor som spelar in i tidsaspekten är (och nu blir jag politiskt inkorrekt) att eleverna i de allra flesta fall inte har de förkunskaper som de nya kursplanerna förutsätter att de ska ha när de kommer till gymnasieskolan – ett problem som kanske löser sig om ett par år (när kursplaner är i fas), men det är ingenting jag skulle satsa några pengar på. Detta innebär att en stor del av tiden går åt till att lappa kunskapshål snarare än att utveckla resultat."

He describes his difficulties in the implementation of GY11. He says that in a few years he hopes (but does not expect: ”det är ingenting jag skulle satsa några pengar på”) that students coming to Swedish secondary schools will be more equipped to meet GY11’s requirements.

After being asked follow up questions (see previous analysis section), Teacher D relates one final thought that is quite telling about how he feels:

"…det kanske finns strategier för att alla ska få komma till tals, men det är svårt just nu."

When reflecting upon how his work is conducted in his classroom, he states that there are other strategies that he could probably employ to involve more students in deep, meaningful discussions, ”but it is difficult right now.” Exactly what it is that is difficult for him right now could be the time he is given, the students’ level of prior knowledge, or just the difficulty in the implementation of new Swedish standards. Regardless of what it is, his statement definitely comes with it a sense of frustration.

**Teacher E** responded that working together with the other teachers in the program (yrkesprogram) would help her students to better understand the meaningfulness of mathematics:
“Samarbete med yrkeslärare skulle, utan tvivel, underlätta för mig att hjälpa eleverna att begripa matematikens meningsfullhet.”

6 Result and discussion

The study was driven by two questions:

1. Does teacher practice match the current research on metacognitive mathematics instruction?
2. Do mathematics teachers feel a need to increase their level of expertise when developing their students’ metacognitive abilities?

In order to answer question 1, a research of the literature was conducted. The literature indicated nine different teaching strategies that can be used to support metacognitive development in mathematics instruction. Interview responses were analyzed/interpreted to determine which of the nine teaching strategies that support metacognitive development were reported:

1. Questioning and questioning strategies
2. Group discussions
3. Think-alouds
4. Summarizing
5. Debriefing
6. Journal writing
7. Guiding self-efficacy
8. Accuracy training
9. Efficient allocation of study time

After analyzing interview responses it can be concluded that the teacher’s reported a limited use of the metacognitive teaching strategies described in the research. Teacher A describes the use of strategies 2, 3, and 8. Teacher B described the use of strategies 1, 2, 3, and 8. Teacher C and D described the use of strategy 2. Teacher E described the use of strategy 7. None of the teachers described the use of strategies 4, 5, 6, or 9. Because the study utilized partially structured interviews to gather data, it cannot be assumed that the teachers do not use these or other strategies, but only that they do not report to the use of them. To determine exactly which strategies the teachers use to support metacognitive development in their classrooms, long-term observational studies would be required.

In order to answer question 2, interview responses were analyzed with regard to the difficulties, possibilities, and suggestions participants gave that would help them in their work toward the learning goal. Teachers A, B, C, and D all report that the amount of time they have for their courses limits their ability to help their students achieve the learning goal. Teachers A and D report that students’ insufficient prior knowledge as a factor that contributes to the task’s difficulty. Teacher B refers to limited access to published national tests as a barrier in his work. Teacher A mentions smaller groups and reliable pre tests as factors that would make the work easier. Teacher E describes that working with other teachers within her program would help in her work.
During the analysis it became evident that interview subjects also repeatedly refer to the difficulty they had in answering the questions. Multiple teachers describe having difficulty in knowing what I wanted them to say or how I wanted them to answer. The questions were purposely posed in a way that was not leading, so that interview subjects had the freedom to answer them in their own way. It was assumed that this tactic would uncover not only the strategies that they use, but also their ability to articulate their understanding of metacognition and how it can be developed in their students. Their difficulty in answering the questions may indicate a need to develop their own declarative metacognitive knowledge of thinking skills, a factor that contributes to their ability to develop higher order thinking skills in their students (Zohar, 1999). Teacher D said, “det är svårt att se vad du egentligen vill att jag ska svara på under de här omständigheterna” (it is difficult to see what you really want me to answer in these conditions). When asked to describe ”de här omständigheterna” he stated that the interview form (email) and vague questions made it difficult for him to know exactly what he was being asked to answer. He said that a face-to-face interview would have allowed him to get quick and simple clarifications so that he may know if he was on the right track. He apologized if this resulted in his responses not being relevant to my questions. This reveals two things. First, the questions were vague and non-leading by design. More developed clarifications would have compromised my ability to gauge the respondents’ knowledge of metacognitive teaching methods and their ability to articulate them. Second, the email interview form was an obstruction in the mind of the respondent. With limited experience in conducting face-to-face interviews as well as not being a native Swedish speaker, on-the-spot requests for clarification may have resulted in over-clarifying thereby limiting the insight to be gained about their current understanding of metacognition.

The analysis also revealed that a number of responses indicated teachers’ emotional states when working toward the learning goal. Teacher B’s responses indicated a level of irritation and/or frustration. Teacher C reported stress. Teacher D’s responses indicated frustration with the implementation of GY11. Stress, irritation, and frustration may come from many places. They all may be seen as an indication that the teacher’s lack a sense of control over what they are able to accomplish. Their responses describe the factors that limit their control over their success (time, prior knowledge, etc). All of the factors described are external (factors outside themselves). Is it possible that there are internal factors (those within themselves) that limit these teachers’ sense of control over their success? Would educating these teachers more extensively in metacognition and the strategies teachers use to develop it give them a greater sense of control? Would this type of education change the teachers’ perception of the time they’re given and how to best utilize it? All of the teachers’ responses were dominated by the discussion of the extensive content they are required to teach and the mathematical methods their students must learn. The time and space they gave to discussing their instructional process and their students’ learning processes was conspicuously subordinate in the responses. It is the researcher’s opinion that integrating metacognitive teaching practices into daily classroom activities as opposed to teaching them after or separately from the acquisition of content knowledge would not only save time, but also encourage deeper more meaningful understanding of the content and inspire and motivate the students to learn more.

To conclude that teachers “feel a need to increase their level of expertise when
developing their students’ metacognitive abilities” is difficult to say without reservation. Interpreting how the interview subjects feel about their own expertise is a sensitive question that requires a high degree of inference on the part of this researcher. However, the responses indicate that the work is difficult and that it sometimes is not prioritized.

The purpose of this research was to study how mathematics instructors develop their students’ metacognitive abilities within mathematics instruction and to study whether these teachers feel adequately prepared to develop their students’ metacognitive abilities. The interviews and analysis with respect to the two guiding questions have revealed the practices these teachers employ and their feelings related to that work. Although indicators of frustration and a lack of control exist it cannot be said that the respondents have reported that they feel a need to increase their level of expertise. However, due to their limited ability to articulate their understanding of metacognition and the strategies that can support it, their need for professional development can be inferred. The results indicate that work needs to be done to improve the scope and frequency of metacognitive teaching practices used in the classroom thereby improving teachers’ level of expertise within metacognition and their ability to articulate it. Improving teachers’ understanding of metacognition and the instructional practices that support its development in mathematics students could do no harm. At the very least it could help with the implementation of GY11.

7 Implications

The results support the comments of Enroth & Nylén (2008) that, although there is substantial literature describing how to support metacognition in the classroom, they did not experience its widespread implementation or awareness of it during their study.

Olsson’s (2011) findings that mathematics teacher’s use of teaching practices that support metacognition were lower than those found in other content are teachers’ practice cannot be confirmed or denied from this research. However, her commentary concerning possible causes for this: that math course plans have not had learning goals that addressed metacognition is now no longer the case. This study has revealed that some of the teachers interviewed in this research have shown signs of stress, frustration, and/or irritation with the work they are now expected to do to help their students’ metacognitive development. These emotions are seen to be a sign of a lack of control over their own possibility for success. This lack of perceived control may be due to the hasty implementation of GY11 or a call for the improvement of teacher education programs to prepare teachers to help their students develop higher order thinking processes demanded by GY11.

Zohar’s (1999) conclusions “that teachers’ intuitive declarative metacognitive knowledge of thinking skills was found to be unsatisfactory for the purpose of teaching higher order thinking in science classrooms” and that “A general practical implication from this finding is that courses which prepare teachers for instruction of higher order thinking should address extensively the issue of declarative metacognitive knowledge of thinking skills” (p. 426) is also supported by this study.
If similar results have been found in and around Stockholm (Enroth & Nylén, 2008), in a suburb of Göteborg (Olsson, 2011), in Israel (Zohar, 1999), and in southern Sweden (present study), this may be an indication of a larger trend. To verify these results more study is needed. This should include long term observational studies of teachers in their classrooms to more carefully and thoroughly map the strategies teachers use to develop metacognition in their students.

As stated in the introduction, the Swedish government has invested over 300 million Swedish crowns to investigate the reasons behind the decline of Swedish students’ mathematics performance and to develop and improve mathematics instruction (Skolverket, 2011). Although there is no “cure all,” this research has found that for these five teachers, and perhaps others like them in Sweden, allocating some of those 300 million crowns to the identification, development, improvement, and implementation of metacognitive teaching strategies for mathematics teachers may be money well spent.
8 References


Schoenfeld, A.H. (1992). Learning to Think Mathematically: Problem Solving,


9 Attachments

9.1 Email interview sent to prospective subjects

Jag heter Don Hill och är lärarstudent vid Högskolan i Halmstad. Efter avslutade studier kommer jag att ha en lärarexamen mot gymnasiet i matematik och psykologi. Parallellt med mina studier arbetar jag som lärare i matematik och engelska på Grundvux i Falkenberg.

Just nu skriver jag på mitt examensarbete, som handlar om hur lärare lägger upp sin matematikundervisning för att eleverna skall nå kunskapskravet:

Eleven kan utvärdera med nyanserade omdömen resultats rimlighet samt valda modeller, strategier, metoder och alternativ till dem (Skolverket, 2011).

Jag hoppas att du vill ställa upp och hjälpa mig genom att beskriva hur du konkret i undervisningen gör för att eleverna skall nå ovanstående kunskapskrav samt vilka hinder och möjligheter som du ser då det gäller att uppnå detta.

För att reda ut ovanstående frågeställningar utför jag e-post intervjuer. Jag har skickat det här mejlet för jag vill veta om just Dina reflektioner om Din egen praktik så att jag får en bild av hur du arbetar med detta med dina elever. Om det är något som jag inte förstår i ditt svar så ber jag att få återkomma för att få förtydliganden. Dina svar på mina frågor kommer att behandlas konfidentiellt d.v.s. att deltagarnas namn inte kommer att avslöjas då resultaten presenteras. Du har också rätt att avsluta ditt deltagande när du vill under intervjunprosessen.

För att förtydliga vad jag vill veta kan kanske dessa följdrågor vara till hjälp:

- Vad gör du i din planering, undervisning och bedömning som du tror kan hjälpa dina elever nå kravet?
- Hur fungerar det för dig? Vilka möjligheter/svårigheter upplever du?
- Vad skulle underlätta för dig för att hjälpa eleverna att nå kravet?

Du får gärna bifoga exempel på planering, lektionsunderlag, bedömningsmoment, etc.

Jag vill även att du lämnar lite bakgrundsinfo. Var snäll och berätta om Din bakgrund som matematik lärare:

- Lärarutbildning eller annan
- Kön
- Skolform Du undervisar i just nu (gymnasieskolan, komvux, osv)
- Antal år du har arbetat som lärare i matematik
- Andra ämnen Du undervisar i
- Nuvarande kurser inom matematik

Tack för ditt deltagande! Var snäll och mejla din respons till mig donhil09@student.hh.se innan den 10:e februari 2012.
Som deltagare har du rätt att läsa mitt arbete när det är färdigt. Indikera med dina svar om du vill ha en kopia när det är färdigt.

Om du har frågor om mitt arbete får du gärna kontakta mig eller min handledare på Högskolan i Halmstad: Monica.Eklund@hh.se