Pricing of exotic options under the Kou model by using the Laplace transform.

Master’s Thesis in Financial Mathematics

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Preface

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Abstract

In this thesis we present the Laplace transform method of option pricing and its realization, also compare it with another methods.

We consider vanilla and exotic options, but more attention we pay to the two-asset correlation options. We chose the one of the modifications of Black-Scholes model, the Kou double exponential jump-diffusion model with the double exponential distribution of jumps, as model of the underlying stock prices development.

The computations was done by the Laplace transform and its inversion by the Euler method. We will present in details proof of finding the Laplace transforms of put and call two-asset correlation options, the calculations of the moment generation function of the jump diffusion by Lévy-Khintchine formulae in cases without jumps and with independent jumps, and direct calculation of the risk-neutral expectation by solving double integral.

Our work also contains the programme code for two-asset correlation call and put options. We will show the realization of our programme in the real data.

As a result we see how our model complies on the NASDAQ OMX Stockholm Market, considering the two-asset correlation options on three cases by stock prices of Handelsbanken, Ericsson and index OMXS30.

Keywords: Double exponential jump-diffusion model, Kou model, Laplace transform, Laplace transform inversion, two-dimensional Euler algorithm, two-asset correlation options.
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Chapter 1

Introduction

The innovations of the financial derivatives are not from so remote past, and now we have already had the growth derivatives markets, more and more such products have been designed and issued by financial institutions. But in connection with this phenomenon, some of those products are complicated. Consequently, there are new problems with the pricing and the hedging of those derivatives.

Nowadays the knowledge about pricing of plain vanilla options is generally used and investors become interested in more complex products such as the exotic options. To study exotic options has become very important because people who deal with options in the market need to be able to price these specific derivatives. In this paper, we focus on the pricing and modeling of such derivative products.

First very crucial revolution in the history of derivatives was done by Fischer Black and Myron Scholes, when they published their groundbreaking paper "The Pricing of Options and Corporate Liabilities" in 1973. Due to this publication theoretically consistent framework for pricing options became available. So far, this model has huge influence on the way that traders price and hedge options. It became the basis of the growth and success of the financial engineering in the last four decades. The idea of this work is based on the assumption, that the asset prices follow a geometric Brownian motion. Till now modifications of this model are commonly used, because of its simplicity and ease of implementation.

Despite the successes of Black-Scholes model (BSM model), and the fact, that the outcomes achieved by this model reflect the real prices quite well, it has some faults, emerged from many empirical investigations, such as the leptokurtic and asymmetric features, the volatility smile and the volatility clustering effect. Also what is evident now, is that in the real world there exist jumps \[ H_5 \], caused by a variety of economical, political and social fac-
tors. According to the observation from the real market prices does not follow a geometric Brownian motion. The Black-Scholes model does not fit real data very well. Many scientists have tried to improve and modify the Black-Scholes model.

Varieties of models have as its object to incorporate the leptokurtic and asymmetric features. Among others there are the chaos theory, fractal Brownian motions, and stable processes [39, 41, 43], [41, 43]; generalized hyperbolic models, including t-model, hyperbolic model and variance gamma model [42, 18, 6]; time changed Brownian motions [25]. Each of them has some advantages, but the problem is that it is hard to get analytical solutions in order to price option, alias they might provide some analytical formulae for standard options, but no fear for interest rate derivatives and exotic options. The variety of models pursue the object to incorporate the "volatility smile".

Among them there are constant elasticity model (CEV) model [13, 14], stochastic volatility and ARCH models [28], a numerical procedure called "implied binomial trees" [53], normal jump models [45]. These models are useful for researches in many fields, such as pricing plain vanilla options, path-dependent options, short maturity options, interest rate derivatives.

Problem is that referred above models may not reflect the leptokurtic and asymmetric features, particularly the "high peak" feature. Because of jump-diffusion issues, in this work we are going to focus on the double exponential jump diffusion model, which is also called Kou model [37].

Besides the crucial properties the double exponential distribution, this model provide following issues:

- It can reproduce leptokurtic and asymmetric features, according to which the distribution of assets return has a higher peak and two heavier tails than the normal distribution, and it is skewed to the left.

- It provides analytical solutions for variety of option pricing problems (plain vanilla options, interest rate derivatives, some exotic options, and options on futures).

- It can reflect the "volatility smile". The fact, that the implied volatility curve looks as "smile" is well-known. The graph of the implied volatility with respect to strike price is a convex curve.

You can read about crucial empirical facts for the pricing models in Chapter 2. Then we consider some modifications of Black-Scholes model in Chapter 3 to illustrate, that there exist several models, but we chose the pricing options under the Kou model, which we describe in Chapter 4 and all algorithm of calculations in detail in this chapter.
We use the Euler algorithm to obtain price of the option by the numerical inversion of Laplace transforms, because the explicit formulas for the inverted Laplace transform in some cases do not exist. Due to fast convergence of Euler algorithm, just a limited number of terms are necessary [18]. Our objective is to consider the double exponential jump-diffusion model and use the Laplace transform for pricing of plain vanilla, barrier and two-asset correlation options. To provide effective method with simple general error bounds we use these methods to confirm the accuracy. We present the Euler algorithm, which can be used for accurate inversion of the Laplace transform of option prices.

And then we apply our research results to valuation of options under the double exponential jump-diffusion model via the Laplace transform in Chapter 5. We are interested to show fruit of our work realized with real data, which we got from the special electronic information system SIX Edge™. For the visualization we wrote our own program in the software for statistical computing R. We will present our results on the example of the NASDAQ OMX Stockholm Market, considering the two-asset correlation options on three cases by index OMXS 30 and stock prices of Handelsbanken, Ericsson. In Chapter 6 we analyze all our results and make a conclusion. Some calculations and the code of our programme you can find in Appendix.
Chapter 1. Introduction
Chapter 2

Empirical facts

A stylized fact is a widely used term referring to empirical results that are so accordant that they are accepted as truth. The stylized findings, which are presented as true, in statistics can only be taken as highly probable. The stylized facts show complex statistics in an easy way. A stylized fact is usually a broad generalization, and due to their generality, they are often qualitative.

**Definition 2.0.1.** If $X$ is a random variable defined on a probability space $(\Omega, F, P)$ with probability density function $f(x)$, then the expected value of $X$ is defined as

$$
E(X) = \int_{\Omega} XdP = \int_{-\infty}^{+\infty} xf(x)dx. \quad (2.1)
$$

For a discrete random variable the expected value is given by

$$
E(X) = \sum_{i} x_{i}p(x_{i}), \quad (2.2)
$$

where $p(x)$ is a probability mass function.

**Definition 2.0.2.** If a random variable $X$ has the expected value $\mu = E(X)$, then the variance of $X$ is given by

$$
Var(X) = E[(X - \mu)^2] = E[(X - E(X))^2] = E(X^2) - [E(X)]^2 \quad (2.3)
$$

**Definition 2.0.3.** The covariance between two real-valued random variables $X$ and $Y$ with finite second moments is

$$
Cov(X, Y) = E[(X - E(X))(Y - E(Y))] = E(XY) - E(X)E(Y). \quad (2.4)
$$
Definition 2.0.4. The correlation $\rho$ between two real-valued random variables $X$ and $Y$ is

$$
\rho = \frac{\text{cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}},
$$

(2.5)
as long as the variances are non-zero \[22\].

Correspondingly for a stock return discrete time series $r(t)$ the autocovariance $\gamma_k$ is defined as

$$
\gamma_k = \text{Cov}(r_t, r_{t-k}) = \text{Cov}(r_t, r_{t+k}),
$$

(2.6)
and the autocorrelation $\rho_k$

$$
\rho_k = \frac{\text{cov}(r_t, r_{t-k})}{\sqrt{\text{Var}(r_t)\text{Var}(r_{t-k})}}.
$$

(2.7)

To estimate sample $\rho_k$ we can use

$$
\hat{\rho}_k = \frac{\sum_{t=k+1}^{T}(r_t - \bar{r})(r_{t-k} - \bar{r})}{\sum_{t=1}^{T} (r_t - \bar{r})^2}.
$$

(2.8)

2.1 The leptokurtic and asymmetric features

Many empirical studies of the real data behavior suggest that the distribution of return has a higher peak and two heavier tails than the normal distribution, especially the left tail. These features are called the leptokurtic and asymmetric features.

Definition 2.1.1. The kurtosis is a measure of the concentration around its mean.The kurtosis is defined as

$$
K = E\left[\left(\frac{x - \mu}{\sigma}\right)^4\right].
$$

(2.9)
The kurtosis minus 3 is also known as excess kurtosis. The distribution is called mesokurtic, if its excess kurtosis equal zero . The normal distribution, which underlies the Black-Scholes model, is the most known instance of a mesokurtic distribution.

If $K > 3$ then the distribution will be called leptokurtic. The leptokurtic distribution will have a higher peak and two heavier tails then normal distribution. Examples of leptokurtic distributions include:

1. double exponential distribution;
2. the family of generalized hyperbolic distributions;
3. the stable distributions.

If $K < 3$ then the distribution is platykurtic. For distributions which are platykurtic, the tails are thinner and they have wider peaks. The uniform distribution is an example of platykurtic distribution.

To estimate kurtosis, we can use

$$\hat{K} = \frac{1}{(n-1)\hat{\sigma}^4} \sum_{i=1}^{n} (X_i - \bar{X})^4,$$

(2.10)

as sample kurtosis, where $\hat{\sigma}$ is the sample standard deviation [37], $\bar{X}$ is the sample mean.

**Definition 2.1.2.** The skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable. The skewness is defined as

$$S = E\left[\left(\frac{x - \mu}{\sigma}\right)^3\right].$$

(2.11)

If the left tail is more pronounced than the right tail, the function is said to have negative skewness. If the reverse is true, it has positive skewness. If the two are equal, it has zero skewness.

The estimation of sample skewness can be done in similar way as in (2.10),

$$\hat{S} = \frac{1}{(n-1)\hat{\sigma}^3} \sum_{i=1}^{n} (X_i - \bar{X})^3.$$

(2.12)

The leptokurtic feature has been noticed since 1950’s. Nevertheless classical finance models simply ignore this feature. For example, according to the Black-Scholes model the stock price is modelled as a geometric Brownian motion, $S(t) = S(0) e^{\mu t + \sigma W(t)}$, where the Brownian motion $W(t)$ has a normal distribution with mean 0 and variance $t$, $\mu$ is the drift and $\sigma$ is the volatility, which is measure of the returns standard deviation. Also in this model the continuously compounded returns, $r(t)$, has a normal distribution, which is an evident contradiction to the leptokurtic feature [37]. As mentioned before the double exponential density function is an example of a leptokurtic distribution, as it has higher peak and tails heavier then the normal distribution.

### 2.2 Implied volatility

The Black-Scholes model uses the different inputs to derive a theoretical value for an option, which depends on a value of the forward realized volatility of
the underlying.
Despite difficulties of the volatility estimation in practice, we definitely know that the option prices are quoted in the market and even if people do not know the volatility, for market system it is known. More precisely, take the Black-Scholes formula for a call, for example, and substitute in the interest rate, the price of the underlying, the exercise price and the time to expiry. Based on the call option price increases monotonically with volatility, there is a one-to-one correspondence between the volatility and the option price. Taking the option price quoted in the market and working backwards, we can say that the market's opinion of the value for the volatility over the remaining life of the option. This volatility, derived from the quoted price for a single option, is called the implied volatility, see [59].

One significant feature of implied volatility is the fact, that if we will compute volatility from a call option with the maturity $T$ and strike $K$ and from a put option with identical parameters, we obtain the same answers due to the put-call parity:

$$S(t) = C(S, K) - P(S, K) + Ke^{-r(T-t)}, \quad (2.13)$$

where $S(t)$ is the asset price at time $t$, $K$ represents the strike price, which is the same for both options, $P(S, K)$ and $C(S, K)$ is the price of the put and call option respectively.

For any possible model, which we can use, the put-call parity must hold to obviate arbitrage, see [59].

If we denote the market price of the call and put like $C_M(S, K)$ and $P_M(S, K)$, while $C_{BS}(S, K)$ and $P_{BS}(S, K)$ will be the call and put prices given by the Black-Scholes formula, then taking the difference between the two equations, we get

$$C_M(S, K) - P_M(S, K) = C_{BS}(S, K) - P_{BS}(S, K). \quad (2.14)$$

From definition of implied volatility we have $\sigma_p(T, K) = \sigma_c(T, K) = \sigma(T, K)$, where $\sigma_c(T, K)$ is implied volatilities derived from the quoted prices in the market for the call option and $\sigma_p(T, K)$ - for the put option. We suppose that the implied volatilities from call and put options with the same maturity $T$ and strike price $K$ must be equal, but in practice it is not the case, because we have bid-ask spreads for options.

Implied volatility has another important singularity that it does not figure as a constant via exercise prices. In other words, if the value of the underlying, the interest rate and the time to expiry are fixed, the price of options via exercise prices should show volatility like an invariable value, but in practice it can not be. And this contradiction exists in some option pricing models. This effect has a reflection in the plot of implied volatility against strike prices, witch holds U-shape and generally called the "volatility smile".
2.3 The volatility clustering effect

Many finance models simply ignore the dependent structure among asset returns and suggest that the stock returns have no autocorrelation. Among them there are capital asset pricing model and Black-Scholes model, etc. Moreover, the assumption, that the stock prices follow “a random walk hypothesis” with independent asset returns underlies most of the classical models. Nevertheless, empirical observations show some interesting dependent structures among asset returns. From this observations researchers have concluded that while asset returns have approximately zero autocorrelation, the volatility of returns are correlated. Whereas returns themselves have almost no correlation, the absolute returns $|Y_t|$ or their squares display a positive, significant and slowly decaying autocorrelation function $\text{Cor}(|Y_t|, |Y_{t+k}|) > 0$ for $k$ ranging from a few minutes to several weeks. This phenomenon is called the volatility clustering effect. Financial models for stock returns with independent increments cannot capture the volatility clustering effect. The jump-diffusion models as special cases of Lévy processes cannot incorporate the volatility clustering effect directly. However, the combination jump-diffusions with stochastic volatilities resulting in the ”affine jump-diffusion models”, see Duffie [16], which can incorporate jumps, stochastic volatility, and jumps in volatility, but these models have another shortcomings.

2.4 The tail behavior

The main purpose for using Lévy processes in finance is the fact that the asset return distributions tend to have tails heavier than those of normal distribution. The tails distribution can be power-type distributions or exponential-type distributions.

In order distinguish the tails distribution we need to consider very large sample size. It can be necessary to use quantiles with very low $p$ values. If the true quantiles have to be estimated from data, then the problem is even worse, as the sample standard deviations need to be considered, resulting in great numbers necessary, see [26]. Therefore, people should be careful to choose a good model based only on the limited empirical data.

For describing power-type distribution, cf. [54], let the right tail of random variable $X$ has a power-type tail if

$$P(X > x) \approx \frac{c}{x^\alpha}, \quad x > 0, \text{as } x \to \infty,$$ (2.15)
the left tail of $X$ has a power-type tail if

$$P(X > -x) \approx \frac{c}{x^\alpha}, \ x > 0, \ as \ x \to \infty. \quad (2.16)$$

By analogy, for exponential-type distributions, see for example [54], we can say that $X$ has a right exponential-type tail if

$$P(X > x) \approx ce^{-\alpha x}, \ x > 0, \quad (2.17)$$

and a left if

$$P(X > -x) \approx ce^{-\alpha x}, \ x < 0, \ as \ x \to \infty. \quad (2.18)$$

However, we can use power type right tail only for discretely compounded models, because there exists one relevant feature of using power type right tail in modeling return distributions. It is that this type can not be used in models with continuous compounding, see [33]. If we consider this case, we will have the infinite value of expected asset price. Note that t-distribution has power type for any degree of freedom.

If we come back to the problem about distinguishing tail distributions we will have next difficulty, it is a risk connected with choosing an acceptable model and preference of using power-type distributions or exponential-type distributions. To measure this risk we can use: Value-at-Risk (or VaR), which is a measure based on quantiles, or the tail conditional expectation [27]. The choice between them depends on whether the measure is used for internal or external risk management.
Chapter 3
Some financial models

The development of modern option pricing models began with Fischer Black and Myron Scholes’ publication in 1973, see [7]. Their work has forever changed the way for both practitioners and theoreticians view of the pricing of derivative securities. The next scientists moved beyond Black-Scholes, making modifications, and now we can read about several option pricing models. In this chapter we consider some the most general of them. Others are left without due attention, but we will mention articles, where you can find their description. There are models, which are conducted to reproduce the volatility clustering effect, such as Stochastic volatility [28], [20] and GARCH models [19]. It worth to mention the affine stochastic-volatility and affine jump-diffusion models, which is combination of stochastic volatility and jump-diffusion models, see [16].

3.1 The Black-Scholes model

The simplicity of Black-Scholes model was achieved by taking following assumptions [59]:

- The price of asset follows a geometric Brownian motion

\[ dS_t = \mu S_t dt + \sigma S_t dW_t, \]  

where \( W_t \) is a Brownian motion, \( \mu \) is the mean value, also known as the drift, \( \sigma \) represent the volatility of the relative price change of the stock price, and \( S_t \) is the current stock price.

- The volatility \( \sigma \) of the stock price change and risk free interest rate \( r \) are assumed to be constant.
• There are no transaction costs associated with hedging a portfolio and no taxes.

• The underlying asset pays no dividends.

• There is no arbitrage opportunities.

• The assets are divisible and short selling are permitted.

• Continuous trading of the underlying is possible.

• Participants can borrow and lend on the risk free interest rate.

Using Itô’s Lemma, we can obtain from \( 3.1 \)

\[
dV = \left( \mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} \right) dt + \sigma S \frac{\partial V}{\partial S} dW , \tag{3.2}
\]

where \( V \) is the price of option. Applying no arbitrage argument, we can get the Black-Scholes partial differential equation \( 59 \)

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 , \tag{3.3}
\]

The solution of the equation \( 3.3 \) is given by \( 24 \)

\[
C(S, t) = SN(d_1) - Ke^{-r(T-t)}N(d_2), \tag{3.4}
\]

\[
P(S, t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1), \tag{3.5}
\]

where \( C \) and \( P \) denote the price of European call and put options respectively, \( (T - t) \) is time to expiration in years, \( K \) is strike price, \( N(\cdot) \) is the cumulative normal distribution function, described by the following formula

\[
N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy , \tag{3.6}
\]

where

\[
d_1 = \frac{\log \frac{S}{E} + (r + \frac{1}{2}\sigma^2)(T - t)}{\sigma \sqrt{T - t}} , \tag{3.7}
\]

\[
d_2 = \frac{\log \frac{S}{E} + (r - \frac{1}{2}\sigma^2)(T - t)}{\sigma \sqrt{T - t}} = d_1 - \sigma \sqrt{T - t} . \tag{3.8}
\]

It is widely recognized that sometimes we can’t get results from the Black-Scholes model, which coincide with the real prices. The reason is that the
Black-Scholes model relies on the idealized assumptions and can not incorporate empirical facts, which we mentioned before. However, this model is a major breakthrough of the option pricing development. Many studies are based on the Black-Scholes model. The main purpose of this studies is to modify classical model and make it more adjusted to the real world. We are going further to consider alternative modifications of the Black-Scholes model.

3.2 Nonlinear chaotic models and a fractional Brownian motion

Let’s consider the evolution of the sequence

\[ h = (h_n) \text{ with } h_n = \ln \frac{S_n}{S_{n-1}}, \]

where \( S_n \) is the level of some price at time \( n \) and

\[ S_n = S_n(\omega) \text{ and } h_n = h_n(\omega) \]

were random variables defined on \((\Omega, \mathcal{F}, (\mathcal{F}_n)_{n \geq 1}, P)\), some filtered probability space, and simulating the statistic uncertainty of real-life situations. For comparing, some deterministic system:

\[ x_{n+1} = f(x_n; X), \]

where \( X \) is a parameter, can produce sequences with behavior similar to that of stochastic sequences [54].

Many economic, including financial, series are actually realizations of chaotic (rather than stochastic) systems. Considering the forecast and predictability of the future price movements, we can say that it has chaotic behavior. We should understand chaos as an explicit lack of order of predictability in asset price behavior. The two required elements of chaos theory are that the prices (some invents) depend on an underlying order, and that even elementary or short events can lead to complex behaviors or events (sensitive dependence on initial conditions).

Fractional Brownian motion was introduced by Kolmogorov, see [31], and studied by Mandelbrot and Van Ness in [40]. In these models where the fractional Brownian motion is instead of the geometrical Brownian motion, see [52], perhaps, arbitrage opportunities exist.
Definition 3.2.1. [54] A continuous Gaussian process $X = (X_t)_{t \geq 0}$, with zero mean and the covariance function equals

$$\text{Cov}(X_t, X_s) = \frac{1}{2} \left( |t - s + 1|^{2H} - 2 |t - s|^{2H} + |t - s - 1|^{2H} \right)$$

(3.9)

is a (standard) fractional Brownian motion with Hurst self-similarity exponent $0 < H \leq 1$.

It has the following properties:

- $X_0 = 0$ and $E(X_t) = 0$ for all $t \geq 0$;
- $X$ has homogeneous increments, i.e.,
  $$\text{Law}(X_{t+s} - X_s) = \text{Law}(X_t), \ s, t \geq 0;$$
  (3.10)

- $X$ is a Gaussian process and $E(X_t^2) = |t|^{2H}, \ t \geq 0$, where $0 < H \leq 1$;
- $X$ has continuous trajectories.

These properties show that a fractional Brownian motion has the self-similarity property. And if Hurst parameter equals $\frac{1}{2}$ the process is a standard Brownian motion.

Definition 3.2.2. [54] A random process $X = (X_t)_{t \geq 0}$ with state space $\mathbb{R}^d$ is self-similar or satisfies the property of (statistical) self-similarity if for each $a > 0$ there exists $b > 0$ such that

$$\text{Law}(X_{at}, t \geq 0) = \text{Law}(bX_t, t \geq 0).$$

(3.11)

In other words changes of the time scale ($t \rightarrow at$) produce the same results as changes of the phase scale ($x \rightarrow bx$).

3.3 Models based on a Lévy process

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space.

Definition 3.3.1. [46] A one-dimensional stochastic process $X = X(t), t \geq 0$ is a Lévy process:

$$X(t) = X(t, \omega), \ \omega \in \Omega,$$

(3.12)

with the following properties:
\[ X(0) = 0 \text{ P-a.s.,} \]

\[ X \text{ has independent increments, that is, for all } t > 0 \text{ and } s > 0, \text{ the increment } X_{t+s} - X_t \text{ is independent of } X_s \text{ and } s \leq t, \]

\[ X \text{ has stationary increments, that is, for all } h > 0 \text{ the increment } X_{t+h} - X_t \text{ has the same probability law as } X_h, \]

\[ X \text{ is stochastically continuous, that is, for every } t > 0 \text{ and } \varepsilon > 0 \text{ then } \lim_{s \to t} P\{|X_t - X_s| > \varepsilon\} = 0, \]

\[ X \text{ has cádlág paths, that is, the trajectories are right-continuous with left limits.} \]

The Lévy process unlike the Brownian motion can have jumps. The jump at time \( t \) is defined by

\[ \Delta X(t) := X(t) - X(t^−). \]

In such type of models the asset price \( S(t) \) is represented as

\[ S_t = S_0 e^{X_t}, \]

where \( X_t \) is the Lévy process.

### 3.4 The constant elasticity of a variance model

The Black-Scholes formula states volatility as constant during the life time of the option. However, from empirical observations of the stock markets we can conclude, that the volatility is increasing with decreasing of stock price. So, stock prices level and the volatility tend to to have negative correlation. Cox and Ross have produced from this effect the model, which is known now like constant elasticity of variance model (CEV) [12], [13]. According to the CEV, the stock price follows the diffusion process

\[ dS_t = \mu S_t dt + \sigma S_t^\beta dW_t, \tag{3.13} \]

where \( \sigma \) is the instantaneous volatility of the stock price return, \( \mu \) is a drift, \( \beta \) is an elasticity parameter (\( 0 < \beta \leq 2 \)) and \( W_t \) is the Brownian motion. The CEV obtains some well-known special cases depending on the choice of \( \beta \). For example, \( \beta = 0 \) indicates a normal distributed asset price, \( \beta = 2 \) yields the classical BSM model. For \( \beta < 2 \) we will have the distribution with increase of the volatility \( \sigma \) increases in response to the decrease of the stock.
price $S$, which is alike leptokurtic feature. The given diffusion process can be reduced to process with constant volatility. Applying Itô formula to the process $x_t = S_t^a / \alpha \sigma$, with $\alpha = 1 - \beta/2$, we obtain

$$dx_t = \frac{S_t^{a-1}}{\sigma} dS_t + \frac{(\alpha - 1)\sigma}{2} dt.$$  \hspace{1cm} (3.14)

According to the definition of the process $x_t$ we have

$$dx_t = \left(x_t \alpha \mu + \frac{(\alpha - 1)\sigma}{2}\right) dt + dW_t.$$  \hspace{1cm} (3.15)

Thereby, we have received a process with constant volatility equal to 1.

### 3.5 An implied binomial tree

Often, price of an option estimated by the Black-Scholes-Merton formula does not coincide with one which is real market price. As Fisher Black said [8], this discrepancy may appear because of one of the following reasons

- The market price is incorrect.
- The parameters used as input for the theoretical value are incorrect.
- The Black-Scholes-Merton theory is incorrect.

In 1994 Dupire, Derman and Kani developed the implied tree model, according to which the market price of option is always correct, see [17], [15]. An arbitrage-free model containing all relevant information from the real market prices is constructed by using information from liquid options with different strikes and maturities.

The process of stock price evolution is discretized in the implied tree model. The volatility is a function of time and the current asset price, which follows random walk given by [24]

$$dS_t = \mu(t)S_t dt + \sigma(S, t)S_t dW_t.$$  \hspace{1cm} (3.16)

The volatility function can be calculated numerically using volatility smile given by the real quoted price of option. The binomial tree pricing method based on the dividing the life of the option into the small time intervals and it is assumed that on the each interval stock price can move in two directions. The risk-neutral valuation principle underlies the way binomial tree approach is used. It implies that the expected return from the traded asset
are assumed to be equal risk-free interest rate and the price of any derivative product is expected return discounted by the risk-free rate. Moreover, this model is preference-free and markets are assumed to be complete. All investors in the risk-neutral world are indifferent to the risk.

3.6 Generalized hyperbolic (GH) models

The modeling of financial assets as stochastic processes is determined by distributional assumptions on the increments and the dependence structure. The returns of most financial assets have semi-heavy tails, i.e. the actual kurtosis is higher than the kurtosis of the normal distribution [39]. Actually, GH distribution is these semi-heavy tails. In this type of models instead of the normal distribution which doesn’t acknowledge the pricing of options by martingale methods, well-known density of GHD distribution is used. Ole E. Barndorff-Nielsen in [6] introduced the GH distributions.

Definition 3.6.1. The one-dimensional general hyperbolic distribution is defined by the following Lebesgue density

\[
gh(x; \lambda, \alpha, \beta, \delta, \mu) = a(\lambda, \alpha, \beta, \delta) \left( \delta^2 + (x - \mu)^2 \right)^{-\frac{3}{2}}
\times K_{\lambda - \frac{1}{2}} \left( \alpha \sqrt{\delta^2 + (x - \mu)^2} e^{\beta (x - \mu)} \right)
\]

\[
a(\lambda, \alpha, \beta, \delta) = \frac{(\alpha^2 - \beta^2)^{\lambda/2}}{\sqrt{2 \pi \alpha^{\lambda - 1/2} \delta \lambda K_{\lambda} (\delta \sqrt{\alpha^2 - \beta^2})}},
\]

where \( K_\lambda \) is a modified Bessel function and \( x \in \mathbb{R} \). The domain of variation of the parameters is \( \mu \in \mathbb{R} \) and

\[
\delta \geq 0, \ | \beta | < \alpha \ \text{if} \ \lambda > 0
\]
\[
\delta > 0, \ | \beta | < \alpha \ \text{if} \ \lambda = 0
\]
\[
\delta \geq 0, \ | \beta | \leq \alpha \ \text{if} \ \lambda < 0
\]

To define the modified Bessel function, let us consider the equation

\[
z^2 \frac{d^2 w}{dz^2} + z \frac{dw}{dz} - (z^2 + v^2) w = 0.
\]

for an arbitrary real or complex \( v \).
Definition 3.6.2. The modified Bessel function of the first kind $I_v(z)$ for $z \geq 0$ and $v \geq 0$ is equal to the solution of Equation (3.19) that is bounded, when $z \to 0$.

The modified Bessel function of the second kind $K_v(z)$ for $z \geq 0$ and $v \geq 0$ is equal to the solution of Equation (3.19) that is bounded, when $z \to \infty$.

The GH distribution and its derived classes ensure fits to the data which are better than normal distribution. Hyperbolic distributions provide an acceptable tradeoff between the accuracy of fit and the necessary numerical effort. But the poor side of GH models is that they are not flexible as respects to different time scales. It will be not so convinient to work with options of various maturities. We need to choose maturity with distribution of asset price as generalized hyperbolic and at another maturities we should compute distributions like convolution powers of first one.

3.7 A time changed Brownian motion and Lévy processes

In models based on time changed Brownian motions and time changed Lévy processes, the asset price $S(t)$ is modeled as

$$S(t) = G(M(t)),$$

as $G$ is a either geometric Brownian motion or a Lévy process, and $M(t)$ is a nondecreasing stochastic process modeling the stochastic activity time in the market. The activity process $M(t)$ may link to trading volumes, see [37].

Definition 3.7.1. Let $X = (X_t)_{t \geq 0}$ denote a stochastic process, sometimes referred to as the base process, and let $T = (T_s)_{s \geq 0}$ denote a nonnegative, nondecreasing stochastic process not necessarily independent of $X$. The timechanged process is then defined as $Y = (Y_s)_{s \geq 0}$, where

$$Y_s = X_{T_s}.$$ (3.20)

In models can be used two methods for the time change:

1. Absolutely continuous time changes. We can say if a setting $T$ is always continuous and $\tau$ shows jumps, this type of time changes has form $T_s = \int_0^s \tau_u \, du$, for an integrable and positive process $\tau = (\tau_s)_{s \geq 0}$. The advantage of this model class is that it leads to affine models which are highly analytically tractable, see [57].
2. Subordinators. We can refer to subordinators a lot of successful financial models in terms of timechanged Brownian motion. It is non-decreasing Lévy processes and therefore is stationary and has independent increments. Subordinators are pure jump processes of possibly infinite activity plus a deterministic linear drift.

3.8 The Merton jump-diffusion model

The main idea of Merton’s jump-diffusion model, presented in 1976 [45], is to embed in to the classical framework of the BSM model discontinuous jump processes. As Merton wrote the total change in the stock price is posited to be the composition of two types of changes

- The "normal" vibrations in price.
- The "abnormal" vibrations in price reflecting to the impact of new information represent jump part.

The model propose, that the change of the underlying asset price is described by

\[ dS = (\alpha - \lambda k)Sdt + \hat{\sigma}SdWt + kdq_t, \]  
\[ (3.21) \]

where $\alpha$ represents the instantaneous anticipated asset price return, $q_t$ is the independent Poisson process, $\hat{\sigma}$ is the instantaneous variance of the return, in case when jumps do not appear; $dW_t$ is a standard Brownian motion. The solution of the stochastic differential equation (3.21) is given by

\[ S(t) = S(0) \exp \left\{ \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma W(t) \right\} \prod_{i=1}^{N(t)} e^{Y_i}, \]  
\[ (3.22) \]

where $N(t)$ is a Poisson process represents the arrival of new information, in other words the events, which influence to the asset price. The model proposes, that the events are independently and identically distributed. According to the Merton, $Y_i, \{i = 1, \ldots, N(t)\}$ is normal distributed. The density function of $Y_i$ is described by

\[ f_{Y_i}(y) = \frac{1}{\sigma' \sqrt{2\pi}} \exp \left\{ - \frac{(y - \mu')^2}{2\sigma'} \right\}, \]  
\[ (3.23) \]

where $\mu'$ is the mean, and $\sigma'$ is the standard deviation of $Y_i$. Despite to the fact, that the Merton model can not reproduce the volatility clustering effect, it can be used for the valuation of the short time options.
3.9 The Kou double exponential jump-diffusion model

The main difference between the Kou model and the Merton model described before is that, the Kou assumed jumps double exponential distributed with the following density function:

\[ f_V(v) = p \cdot \eta_1 e^{-\eta_1 v} \mathbb{1}_{v \geq 0} + q \cdot \eta_2 e^{\eta_2 v} \mathbb{1}_{v < 0}. \] (3.24)

Here \( \eta_1 > 1; \quad \eta_2 > 0; \quad p, q \geq 0; \quad p + q = 1; \) \( p \) is the probability of the upward jumps and \( q \) is the probability of the downward jumps. In comparison with Merton model, where just one random variable reflects bad and good news, the double exponential jump diffusion model has better economical interpretation, due to the distribution of the jumps.

We are going to consider the Kou model more in details in the next chapter.
Chapter 4

The Kou model and option pricing

4.1 Introduction to option pricing

The usage of options to ensure economic security or for return dates deeply back in our history. The first published account of options use was in Aristotle’s Politics, published in 332 B.C., it said that the creator of options was some great philosopher, astronomer and mathematician, Thales. Then options appeared again during the famous Tulip mania of 1636 (the tulip bulb options).

By the mid 1900’s, the Put and Call Brokers and Dealers Association was formed, but the public acceptance was limited, and options needed an illiquid investment vehicle as they were not standardized and could not be exercised until their expiration dates. Then because of people had already recognized the real utility of these contracts, and though the long developments in the end of the 18 century option contracts were finally accessible to the general population and were ready to flourish. Gradually, already standardized option contracts were provided the guarantor (Options Clearing Corporation(OCC)), the market maker system was established for creating and ensuring a two-sides market with the best bids and offers for public customers. And when people have had an affective infrastructure, they encountered the next problem: you might stand ready to trade a particular option, but only at a fair price. The general method of evaluating of options was essential, because pricing was random.

Fisher Black and Merton Sholes solved this problem in \[7\] around the same time as establishment of the Chicago Board of Exchange (CBOE) when they developed a mathematical formula for calculating the theoretical value of a
Chapter 4. The Kou model and option pricing

The Black-Scholes model gives the value of the determined option by the strike price, the underlying asset, the time to expiration, the interest rate, and the volatility of the underlying asset. It gave opportunity for the individual trader to trade by using the same computations that profession traders use and also opened the business of option trading to a wide public, like a big progress of the whole financial market.

In 1975-1977, with exchange-traded options increasing in demand, the stock exchanges began trading call options such as American, Philadelphia, and Pacific stock exchange. Put options were created. As people became more interested in options contracts and more researchers started to research pricing and trading of them.

Today, derivatives are used to hedge against the risk in different spheres. For instance banks can use derivatives to abate the risk that the short-term interest rates will increase and abate the profit, which have the investors on fixed interest rate securities and loans, farmers can use derivatives against the falling prices of harvest before they will realized results of their work in the market. Pension funds use derivatives to insure against big falls in the value of the portfolios, and insurance companies make credit derivatives by selling credit protection to security institutions and banks. Traders use interest rate swaps, options and swaptions to hedge for reducing the prepayment risk associated with home mortgage financing. Electricity producers hedge to reduce changes unappropriated for the season.

Besides the risk management derivative markets act the important place in pricing traded items in the markets, and then distributing those prices as information throughout the economy and the market. Therefore these prices are significant not only for selling and buying but also using and producing in other markets and it is reflected in the height of commodity and security prices, interest rates and exchange rates.

Nowadays trader has a very big variety of choice for making portfolio. In general, he can select options on two categories:

**Plain vanilla** it can be European or American, call or put, bond option, warrants. They have standard well-defined properties, their prices are valued by traders on a regular basis.

**Exotic** the options with complex financial structures: Asian, barrier, binary, compound, lookback, chooser, Russian. They are nonstandard products that have been created by financial engineers. These products are usually a small part of portfolio, but it is very relevant for traders, because they are, in general, more profitable than plain vanilla.
It is just examples, but in the market there are a lot of different styles and the new ones often appear, see [30]. Some of them is widespread and actively traded, like European, American, some of them are complicated and we can trade only in special case, like Russian, for example.

In our thesis we will pay your attention to the rare type of the option - two asset correlation options (with two underlying assets and two strike prices). The payoff for call is \( \text{max}(S_1 - X_1; 0) \) if \( S_2 > X_2 \) and 0 otherwise and for put is \( \text{max}(X_1 - S_1; 0) \) if \( S_2 < X_2 \) and 0 otherwise. In [61] Zhang proposed the formulas by which we can price these type of options:

\[
C = S_2 e^{(b_2 - r)T} M(y_2 + \frac{\sigma_2 \sqrt{T}}{\rho}, y_1 + \frac{\rho \sigma_2 \sqrt{T}}{\rho}) - X_2 e^{-rT} M(y_2, y_1; \rho) \quad (4.1)
\]

\[
P = X_2 e^{-rT} M(-y_2, -y_1; \rho) - S_2 e^{(b_2 - r)T} M(-y_2 - \frac{\sigma_2 \sqrt{T}}{\rho}, -y_1 - \frac{\rho \sigma_2 \sqrt{T}}{\rho}), \quad (4.2)
\]

where \( M(a, b; \rho) \) is the two-dimensional normal distribution with the correlation coefficient \( \rho \) between the returns on the two assets and

\[
y_1 = \frac{\ln(S_1/X_1) + (b_1 - \sigma_1^2/2)T}{\sigma_1 \sqrt{T}} \quad (4.3)
\]

\[
y_2 = \frac{\ln(S_2/X_2) + (b_2 - \sigma_2^2/2)T}{\sigma_2 \sqrt{T}} \quad (4.4)
\]

During a long time people have tried to research these financial instruments, because for successful strategy they need to be able to price these specific derivatives. And they have already got the crucial results. We have mentioned some models for option pricing. But many of them are not extended for exotic options, only for vanilla options, because it is more difficult to provide some model or method which will compute the option price for contracts with complicated structure. In our thesis we will present one good way - the application of the Laplace transform in option pricing.

### 4.2 The motivation of the choice of the Kou model

The Kou model possess some crucial features, which can be useful for the option pricing. Firstly, the Kou model is internally self-consistent, which means that this model is arbitrage-free and can be embedded in an rational expectations equilibrium setting, [37]. Moreover, the considered model can incorporate two of the empirical facts mentioned before. Thus the Kou model overcomes the problems, which was
faced by the BSM model. It reproduces leptokurtic and asymmetric feature the "volatility smile" effect. Nevertheless, the empirical tests should not be judgement whether model is good or not. The models with many parameters usually reflect the empirical facts better, but these models have less tractability on the other hand. This is one of the explanation why the BSM is still so popular among traders. The Kou model retains analytical tractability, it yields closed-form solutions not only for standard plain vanilla options, but also for variety of exotic options, such as lookback options, barrier options, perpetual American options, see [35], [36]. According to the Ramezani and Zeng [50], that the double exponential jump-diffusion model fits real data better than the Merton’s normal jump-diffusion model, and both of them fit the data better than the BSM model. So, the double exponential jump-diffusion model as a modification of BSM has improved the classical model, and it is tractable.

Furthermore, there are two more crucial features of the Kou model provided by the double exponential distribution of jumps. Firstly, it is the memoryless property. This feature is the reason why it is feasible to obtain the closed-form solutions for various option pricing problems, such as lookback, barrier and perpetual American options under the Kou model, while it is quite difficult using many other models, even Merton model, see [37]. Indeed, the double exponential distribution of jumps provides some economical interpretation to the model. It is known from behavioral finance, that the market has both overreaction and underreaction to the arrival of new information. The jump part can indicate the impact of the news to the market performance of the stock price, while a geometric Brownian motion corresponds to the absence of the outside news. The asset price changes in response to the arrival of the positive and negative information, which appears according to a Poisson process, while the amount rate of the change conform to the jump size distribution, cf. [37].

The refereed above properties are attractive for the practitioners and can persuade market participants to switch from the classical BSM model to the double exponential jump-diffusion Kou model.

Since the double exponential jump diffusion model is a special case of Lévy processes, the stock returns have independent increments. In fact, it means that the Kou model can not incorporate the the volatility clustering effect. It explains that the Kou model as the option pricing model does not perform better than BSM model in case of a highly volatile underlying stock. Nevertheless, the combination of the Kou model and another processes, such us stochastic volatility model, can overcome the considered shortcoming.
4.3 The model description

The Kou model assumes that the underlying asset price consists of two parts, one is continuous part represented by a geometric Brownian motion and the other a jump part, with the double exponential distributed jump sizes and Poisson distributed jump times. The stock price is modeled as

\[
\frac{dS(t)}{S(t-)} = \mu dt + \sigma dW(t) + d \left( \sum_{i=1}^{N(t)} (Z_i - 1) \right),
\]

(4.5)

where \( W(t) \) denotes a Brownian motion, \( N(t) \) represents a Poisson process with rate \( \lambda \), and \( \{Z_i\} \) is a sequence of independent identically distributed nonnegative random variables, which are the jump sizes, \( \mu \) is the drift, \( \sigma \) denotes volatility, cf. [37]. The model assumes that the all sources of randomness, \( N(t) \), \( W(t) \) and the jump sizes are independent. The model can provide the analytical solution even for complex option pricing problems, due to the simplifying assumptions that the drift and the volatility are constant, while the jumps and the Brownian motion are one-dimensional.

The stochastic differential equation (4.5) can be solved by using Itô’s formula, see [11].

\[
S(t) = S(0) \exp \left\{ \left( \mu - \frac{1}{2}\sigma^2 \right) t + \sigma W(t) \right\} \prod_{i=1}^{N(t)} Z_i.
\]

(4.6)

Kou showed that under the double exponential jump diffusion model the rational-expectations equilibrium price of an option equals the expectation of the discounted option payoff under a risk-neutral measure \( P^\star \), see [33]. Under this measure, the logarithmic return process \( Y(t) = \log(S(t)/S(0)) \) is given by Kou et al., see [36],

\[
Y(t) = (r - \delta - \frac{1}{2}\sigma^2 - \lambda \zeta) t + \sigma W(t) + \sum_{i=1}^{N(t)} V_i,
\]

(4.7)

where \( r \) is the risk-free rate, \( \delta \) is the continuous dividend yield, \( \zeta = E^\star[\exp Y] - 1 = p \eta_1 / (\eta_1 - 1) + q \eta_2 / (\eta_2 + 1) - 1 \), and \( V_i = \log(Z_i) \) is double exponential distributed with the density function given by

\[
f_V(v) = p \cdot \eta_1 e^{-\eta_1 v} 1_{v \geq 0} + q \cdot \eta_2 e^{\eta_2 v} 1_{v < 0},
\]

(4.8)

where \( p \) and \( q \) are the probability of the upward and downward jumps respectively and \( p + q = 1 \). There are also restrictions \( \eta_1 > 1 \); \( \eta_2 > 0 \). As Maekawa described [38], that the \( V \) can be represented like random variable, which can either take value \( \xi^- \) with probability \( p \) and the mean \( \eta_1^{-1} \) or \( \xi^+ \) with probability \( q \) and the mean \( \eta_2^{-1} \).
4.4 The Laplace transform for the option pricing

The Laplace transform of a function $f(t)$, defined for all real numbers $t \geq 0$, is the function $\hat{f}(\xi)$, defined by

$$\hat{f}(\xi) = L(f(t)) = \int_0^\infty \exp(-\xi t)f(t)\,dt,$$

such that the parameter $\xi$ is a complex number $\xi = \sigma + i\omega$, cf. [58].

In order to price the option under the Kou model we will use the Laplace transform, widely known as the effective mean for evaluating the derivative products. The double exponential jump diffusion model implies that the price of path-dependent options as well as the standard options can be calculated numerically by using Laplace transforms.

In the survey of Kou [37], the price of a European call option with maturity $T$ and strike price $K$ is given by

$$C_T(k) = e^{-rT}\mathbb{E}^*[\max(S(T) - K, 0)],$$

where $k = -\log K$.

The price of a European put option is given in the similar way

$$P_T(k') = e^{-rT}\mathbb{E}^*[\max(K - S(T), 0)],$$

where $k' = \log(K)$. We now consider the double exponential jump diffusion model, cf. [32.22] and [32.24]. For further calculations we need to introduce the moment generating function of the return process $Y(t)$, which is

$$\mathbb{E}^*[e^{\theta Y(t)}] = e^{G(\theta)t},$$

where the function $G(x)$ is defined as

$$G(x) = x\tilde{\mu} + \frac{1}{2}x^2\sigma^2 + \lambda(E[e^{xV}] - 1),$$

where $\tilde{\mu} = r - \delta - \frac{1}{2}\sigma^2 - \lambda\zeta$ under the risk-neutral measure and $r$, $\delta$, $\lambda$, $\zeta$ are defined as in [4.7]. As Kou and Wang showed [34], the equation $G(x) = a$ for positive values of $a$ has exactly four roots $\beta_{1,a}$, $\beta_{2,a}$, $-\beta_{3,a}$, $-\beta_{4,a}$, such that $0 < \beta_{1,a} < \eta_1 < \beta_{2,a} < \infty$ and $0 < \beta_{3,a} < \eta_2 < \beta_{4,a} < \infty$. The Laplace
transform of the European call and put options can be derived with respect to $k$ and $k'$, respectively, cf. [36],

\[
\hat{f}_C(\xi) : = \int_{-\infty}^{+\infty} e^{-\xi k} C_T(k) dk = e^{-rT} \frac{S(0)^{\xi+1}}{\xi(\xi + 1)} e^{G(\xi+1)T}, \quad \xi > 0, \quad (4.14)
\]

\[
\hat{f}_P(\xi) : = \int_{-\infty}^{+\infty} e^{-\xi k'} P_T(k') dk' = e^{-rT} \frac{S(0)^{-(\xi-1)}}{\xi(\xi - 1)} e^{G(-(\xi-1))T}, \quad \xi > 1. \quad (4.15)
\]

As we can see the Laplace transform can be applied to the standard plain vanilla options. Now our aim is to check how it will work in the case of the path-dependent options. Many models can give accurate results for standard options pricing problems, but the inversion of the Laplace transform under the Kou model can provide us solution for variety of the more complicated exotic options. In order to study the pricing of the exotic options, such as barrier and lookback options, under the double exponential jump diffusion model, it is important to understand first passage time.

**Definition 4.4.1.** [37] The first passage time of a jump-diffusion process $Y(t)$ to a flat boundary $b$ is defined by

\[
\tau_b := \inf \{ t \geq 0; Y(t) \geq b \}, \quad b > 0,
\]

where $Y(\tau_b) := \limsup_{t \to \infty} Y(t)$, on the set $\{\tau_b = \infty\}$.

The closed-form solution for the barrier and lookback options under the BSM model was achieved by Merton [44] and Goldman [21]. It was showed by Kou and Wang [34], that for the Kou model the exponential distribution memoryless property leads to

- The conditional memoryless property of the jump overshoot.
- The conditional independence of the overshoot having non-zero positive value, $Y_\tau - b$, and the first passage time $\tau_b$.
- Analytical solution for the Laplace transforms of $\tau_b$. 
Let us consider the pricing of the path-dependent options under the Kou model, for example an up-and-in call option (UIC). The pricing of the other types of barrier options can be done in similar way using the symmetries, see [23]. An UIC is a option, which will be triggered if the path of the asset price cross the certain price level above, which is called barrier. In the paper of Kou and Wang [34], the price of an UIC option is defined by

\[
UIC(k,T) = E^*\left[e^{-rT}(S(T) - e^{-k})^+ \mathbb{1}_{\{\tau_b < T\}}\right],
\]

(4.17)

where \(K\) denotes the strike price, \(k = -\log(K)\) is the transformed strike price, \(H > S(0)\) denotes the barrier level, \(b = \log(H/S(0))\) is transformed barrier. The problem is that the one-dimensional Laplace transform of UIC involves many special functions, which is complicated for computation. In 2005 the two-dimensional Laplace transform for (4.17) was showed, see [36], which made the formula more convenient for calculations.

**Theorem 4.4.1.** [36]

For \(\xi, \alpha\) such that \(0 < \xi < \eta_1 - 1\) and \(\alpha > \max(G(\xi + 1) - r, 0)\), (such a choice of \(\xi\) and \(\alpha\) is possible for all small enough \(\xi\) as \(G(1) - r = -\delta < 0\)), the Laplace transform with respect to \(k\) and \(T\) of UIC\((k,T)\) is given by

\[
\hat{f}_{UIC}(\xi,\alpha) = \int_0^\infty \int_{-\infty}^{\infty} e^{-\xi k - \alpha T} UIC(k,T) dk dT
\]

\[
= \frac{H^{\xi+1}}{\xi(\xi+1)} \frac{1}{r + \alpha - G(\xi + 1)} \left( A(r + \alpha) \frac{\eta_1}{\eta_1 - (\xi + 1)} + B(r + \alpha) \right),
\]

(4.18)

where

\[
A(h) : = E^*[e^{-h \tau_1} \mathbb{1}_{\{X(\tau_1) > h\}}] = \frac{(\eta_1 - \beta_{1,h})(\beta_{2,h} - \eta_1)}{\eta_1(\beta_{2,h} - \beta_{1,h})} [e^{-b\beta_{1,h}} - e^{-b\beta_{2,h}}],
\]

(4.19)

\[
B(h) : = E^*[e^{-h \tau_2} \mathbb{1}_{\{X(\tau_2) = h\}}] = \frac{\eta_1 - \beta_{1,h}}{\beta_{2,h} - \beta_{1,h}} e^{-\beta_{1,h}} + \frac{\beta_{2,h} - \eta_1}{\beta_{2,h} - \beta_{1,h}} e^{-\beta_{2,h}},
\]

(4.20)

with \(b = \log(H/S(0))\), and \(\beta_{1,h}, \beta_{2,h}\) are two positive roots of the equation \(G(x) = h\).
According to Petrella [48], the Laplace transform of the UIC option under the BSM model with respect to $k$ and $T$ is given by

$$\hat{f}_{UIC}(\xi, \alpha) = e^{-rT} E^*\left[e^{-\alpha \tau_b} \frac{X(H/X)^{(\xi+1)}}{\xi(\xi+1)(\alpha - G(\xi + 1))}\right], \quad (4.21)$$

where $\tau_b = \inf\{t \geq 0, Y_t \geq b\}$, $b = \ln(H/S)$, and $G(\zeta) = \zeta \mu + (\zeta \sigma)^2/2$.

We also can get the Laplace transform for the standard lookback option. Let’s consider the put option, while derivation of the Laplace transform for the lookback call can be done in the similar way. The price of the considered lookback put option is given by [36]

$$LP(T) = E^*\left[e^{-rT}\max\left\{M, \max_{0 \leq t \leq T} S(t)\right\} - S(t)\right] = E^*\left[e^{-rT}\max\left\{M, \max_{0 \leq t \leq T} S(t)\right\} - S(t)\right], \quad (4.22)$$

where $M \geq S(0)$ is the prefixed maximum at time 0.

The Laplace transform of the lookback put with respect to the time to maturity $T$ for any positive value of $\xi$ is given by [35]

$$\int_0^\infty e^{-\alpha T} LP(T) dT = \frac{S(0)A_\alpha}{C_\alpha} \left(\frac{S(0)}{M}\right)^{\beta_{1,a+r} - 1} + \frac{S(0)B_\alpha}{C_\alpha} \left(\frac{S(0)}{M}\right)^{\beta_{2,a+r} - 1} + \frac{M}{\alpha + r} - \frac{S(0)}{\alpha}, \quad (4.23)$$

where

$$A_\alpha = \frac{(\eta_1 - \beta_{1,a+r})\beta_{2,a+r}}{\beta_{1,a+r} - 1},$$

$$B_\alpha = \frac{(\beta_{2,a+r} - \eta_1)\beta_{1,a+r}}{\beta_{2,a+r} - 1},$$

$$C_\alpha = (\alpha + r)\eta_1 (\beta_{2,a+r} - \beta_{1,a+r}),$$

and $\beta_{1,a+r}, \beta_{2,a+r}$ are two positive roots of the equation $G(x) = \alpha + r$.

### 4.5 The Euler algorithm and it’s merits

Sometimes it is not easy to obtain any explicit analytical solutions for the probabilistic model, therefore we should use some numerical algorithm. Many numerical methods have been developed in the last decade. We present the Euler Laplace transform inversion algorithm, but there exist others, such that:
1. Lattice methods are the most popular and general methods, see [29].

2. Monte Carlo simulation is also one of the general methods, see [9].

These methods can require a lot of programming effort and can take a lot of time to produce accurate result.

3. The Gaver-Stehfest method. It requires the high numerical accuracy, see [56]. For comparing, the EU algorithm requires a precision of 12 digits and the Gaver-Stehfest method needs 80 digits accuracy. And the results of observation by Petrella, see [48], show that the EU method is more robust than Gaver-Stehfest method.

4. Fourier transform inversion. In some cases it is difficult to derive the formulae, see [4].

By using the Euler algorithm (EA) we can invert functions defined on positive real line, with extension of the algorithm - to the entire real line, see [48], and it is very useful, because it is likely to made Laplace transform and invert log-strike price by EA. The method, which we have chosen, has the following advantages:

- It provides error bounds.
- The speed of convergence is very high.
- Comparing with other methods, it does not require a big numerical accuracy.

4.6 The Euler method for the Laplace transform inversion

The Euler method for the Laplace transform inversion is first described in [1]. It makes use of the Euler summation, cf. [55], which speed up the summation in the algorithm. Now we will present the formulas of the inversion Laplace transform, bounds on the one-dimensional and two-dimensional discretization error.

For a real function \( f(\cdot) \) defined in \( R \), for any \( t \neq 0 \)

\[
f(t) = \frac{e^{\frac{3}{2}}}{2t} Re\left( \hat{f}\left(\frac{A}{2t}\right) \right) + \frac{e^{\frac{3}{2}}}{t} \sum_{j=1}^{\infty} (-1)^j Re\left( \hat{f}\left(\frac{A+2j\pi i}{2t}\right) \right) - e_d, \tag{4.24}
\]
where \( \hat{f}(\cdot) \) is the Laplace transform of \( f(\cdot) \) with respect to a logarithm of the strike, and \( A \) is a positive constant, which was found by the experiments, \( t \) is equal \( \ln(K/X) \), \( K \) is strike price and \( X = S \cdot e^{-k} \) and \( e_d = e_d^+ + e_d^- \), with

\[
e_d^+ = \sum_{j=1}^{\infty} e^{-jA}f((2j+1)t), \quad (4.25)
\]

\[
e_d^- = \sum_{j=-\infty}^{-1} e^{-jA}f((2j+1)t), \quad (4.26)
\]
is the discretization error, see [48].

For extending to the two-dimensional EA, suppose a real function \( f(t_1, t_2) \) defined in \( \mathbb{R}^2 \) (or in \( \mathbb{R}^+ \times \mathbb{R} \)), where \( \hat{f}(\cdot, \cdot) \) is the Laplace transform of \( f(\cdot, \cdot) \), \( A_1, A_2 \) are arbitrary constants, \( l_1, l_2 \) are integers that are used to control the round-off error, then

\[
f(t_1, t_2) = \frac{\exp(A_1/(2l_1) + A_2/(2l_2))}{4t_1^l_1 t_2^l_2} \times \left( \hat{f}\left( \frac{A_1}{2l_1 t_1}, \frac{A_2}{2l_2 t_2} \right) \right.
\]
\[
+ 2 \sum_{j_1=1}^{l_1} \sum_{j_2=0}^{\infty} (-1)^j \Re \left[ \sum_{k_1=1}^{l_1} \sum_{k_2=0}^{\infty} (-1)^k \exp \left( -\frac{(ij_1 \pi}{l_1} + \frac{ik_1 \pi}{l_2} \right) \right] 
\]
\[
\times \left( \frac{A_1}{2l_1 t_1} - \frac{ij_1 \pi}{l_1} \right) \times \hat{f}\left( \frac{A_1}{2l_1 t_1} - \frac{ij_1 \pi}{l_1}, \frac{A_2}{2l_2 t_2} + \frac{ik_1 \pi}{l_2} \right) \right)
\]
\[
+ 2 \sum_{j_1=1}^{l_1} \sum_{j_2=0}^{\infty} (-1)^j \Re \left[ \sum_{k_1=1}^{l_1} \sum_{k_2=0}^{\infty} (-1)^k \exp \left( -\frac{(ij_1 \pi}{l_1} - \frac{ik_1 \pi}{l_2} \right) \right] 
\]
\[
\times \hat{f}\left( \frac{A_1}{2l_1 t_1} - \frac{ij_1 \pi}{l_1} - \frac{ik_1 \pi}{l_2} \right) \right)
\]
\[
\left( \hat{f}\left( \frac{A_1}{2l_1 t_1} - \frac{ij_1 \pi}{l_1}, \frac{A_2}{2l_2 t_2} + \frac{ik_1 \pi}{l_2} \right) \right), \quad (4.27)
\]

\( e_d \) represents the discretization error for a function defined in \( \mathbb{R}^2 \),

\[
e_d = e_d^{++} + e_d^{+-} + e_d^{-+} + e_d^{--}, \quad \text{with} \quad (4.28)
\]

\[
e_d^{++} = \sum_{j_1=1}^{+2} e^{-j_1A_1-j_2A_2}f((2j_1+1)t_1, (2j_2+1)t_2), \quad (4.29)
\]
\[ e_d^{++} = \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} e^{-j_1 A_1-j_2 A_2} f((2j_1 + 1)t_1, (2j_2 + 1)t_2), \]  
\[ e_d^{+-} = \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} e^{j_1 A_1-j_2 A_2} f((-2j_1 + 1)t_1, (2j_2 + 1)t_2), \]  
\[ e_d^{-+} = \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} e^{j_1 A_1+j_2 A_2} f((-2j_1 + 1)t_1, (-2j_2 + 1)t_2), \] 
where
\[ \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} = \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty}. \]

### 4.7 The realization of the Euler method for the Laplace transform inversion on a two-asset correlation option

Here we present the realization of the two-dimensional EA for the Laplace transform inversion on a two-asset correlation option, when the underlying stock prices are assumed to follow a two-dimensional geometric Brownian motion with correlation equals to \( \rho \) as well as the Kou jump diffusion process. Suppose the functions \( P_\rho \) and \( C_\rho \) defined in \( \mathbb{R} \times \mathbb{R} \), which have values, see [48],

\[ P_\rho^T(K_1, K_2) = e^{-rT} E^*[(K_1 - S_T^{(1)})^+ \mathbb{1}_{\{S_T^{(2)} < K_2\}}] \]
\[ = e^{-rT} E^*[(K_1 - S_0^{(1)} e^{Y_{1T}})^+ \mathbb{1}_{\{S_0^{(2)} e^{Y_{2T}} < K_2\}}], \]  
\[ (4.33) \]

\[ C_\rho^T(K_1, K_2) = e^{-rT} E^*[(S_T^{(1)} - K_1)^+ \mathbb{1}_{\{S_T^{(2)} > K_2\}}] \]
\[ = e^{-rT} E^*[(S_0^{(1)} e^{Y_{1T}} - K_1)^+ \mathbb{1}_{\{S_0^{(2)} e^{Y_{2T}} > K_2\}}], \]  
\[ (4.34) \]

where \( S_0^{(i)}, Y_{1T}^{(i)} \) are respectively the initial value and the return process of asset \( i \) and \( K_i \) is strike relative to the \( i \)th asset, for \( i = 1, 2 \). The two processes \( Y_{1T}^{(1)} \) and \( Y_{1T}^{(2)} \) are correlated. Let \( X_i < S_i \) be rescaling factors and \( k_i = \log(K_i/X_i) \) with \( i = 1, 2 \) for a put option and \( X_i < K_i \) be rescaling factors and \( k_i = -\log(K_i/X_i) \) with \( i = 1, 2 \) for a call option. Then

\[ P_\rho^T(k_1, k_2) = e^{-rT} X_1 E^*\left[\left(e^{k_1} - \frac{S_0^{(1)}}{X_1} e^{Y_{1T}^{(1)}}\right)^+ \mathbb{1}_{\{(S_0^{(2)}/X_2) e^{Y_{2T}^{(2)}} < e^{k_2}\}}\right], \]  
\[ (4.35) \]
Theorem 4.7.1. The Laplace transform of $P^p_T(k_1, k_2)$ in $(k_1, k_2)$ is given by, see [47].

$$
\hat{f}_p(\xi_1, \xi_2) = e^{-rT} X_1 \left( \frac{S_0^{(1)}}{X_1} \right)^{-(\xi_1-1)} \left( \frac{S_0^{(2)}}{X_2} \right)^{-\xi_2} \frac{1}{\xi_1(\xi_1 - 1)\xi_2} E^* \left[ e^{-(\xi_1-1)Y^{(1)}_T} e^{-\xi_2 Y^{(2)}_T} \right].
$$

(4.37)

The Laplace transform of a two asset correlation call option $C^p_T(k_1, k_2)$ is

$$
\hat{f}_c(\xi_1, \xi_2) = e^{-rT} X_1 \left( \frac{S_0^{(1)}}{X_1} \right)^{(\xi_1+1)} \left( \frac{S_0^{(2)}}{X_2} \right)^{\xi_2} \frac{1}{\xi_1(\xi_1 + 1)\xi_2} E^* \left[ e^{(\xi_1+1)Y^{(1)}_T} e^{\xi_2 Y^{(2)}_T} \right],
$$

(4.38)

where $r$ is the interest rate, $T$ is time to maturity, $S_0^{(1)}$ and $S_0^{(2)}$ are the spot prices and $X_1, X_2$ are rescaling factors. For jump diffusion processes the underlying return processes are

$$
Y^{(i)}_T = (r - \sigma_i^2 / 2)T + \sigma_i W^{(i)}_T + \sum_{j=1}^{N_i(T)} V^{(i)}_j, \quad i = 1, 2
$$

(4.39)

where $W^{(i)}_T$ are the standard Brownian motions, $N_i(T)$ are Poisson processes with rate $\lambda T$ and $\rho = \text{Corr}(W^{(1)}_T, W^{(2)}_T)$ in the case of uncorrelated jumps.

We will show the proof in the case of a put option Laplace transform.

Proof(Put case). The price of two-asset correlation put option is given by

$$
P^p_T(K_1, K_2) = e^{-rT} E^*[(K_1 - S^{(1)}_T)^+ \mathbb{1}_{(S^{(2)}_T < K_2)}],
$$

(4.40)

where $K_1, K_2$ are the strike prices and $S^{(1)}_T, S^{(2)}_T$ are the stock prices at time $T$. $E^*[]$ is the expectation under the risk-neutral measure, [61].

Put $k_1 = \log(K_1/X_1)$ and $k_2 = \log(K_2/X_2)$, where $X_1, X_2$ are rescaling factors $X_i < S_i, \ i = 1, 2$. The Laplace transform is

$$
\hat{f}_p(\xi_1, \xi_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\xi_1 k_1 - \xi_2 k_2} e^{-rT} E^*[(K_1 - S^{(1)}_T)^+ \mathbb{1}_{(S^{(2)}_T < K_2)}] dk_1 \ dk_2
$$

$$
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-rT} e^{-\xi_1 k_1 - \xi_2 k_2} X_1 E^* \left[ (e^{k_1} - \frac{S^{(1)}_T}{X_1})^+ \mathbb{1}_{(S^{(2)}_T/X_2 < e^{k_2})} \right] dk_1 \ dk_2.
$$
The Fubini theorem justifies a change of the integration order and we get,

\[
\hat{f}_p(\xi_1, \xi_2) = e^{-rT}X_1 E^* \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\xi_1 k_1} \left( e^{k_1} - \frac{S_T^{(1)}}{X_1} \right) + e^{-\xi_2 k_2} \mathbb{I}_{\{k_2 > S_T^{(2)}/X_2\}} \right] dk_1 dk_2 
\]

\[
= e^{-rT}X_1 E^* \left[ \int_{\log(S_T^{(2)}/X_1)}^{\infty} e^{-\xi_1 k_1} \left( e^{k_1} - \frac{S_T^{(1)}}{X_1} \right) dk_1 \int_{\log(S_T^{(2)}/X_2)}^{\infty} e^{-\xi_2 k_2} dk_2 \right] 
\]

\[
= e^{-rT}X_1 E^* \left[ \frac{1}{\xi_1(\xi_1 - 1)} \left( \frac{S_T^{(1)}}{X_1} \right)^{-\xi_1 + 1} \frac{1}{\xi_2(\xi_2 - 1)} \left( \frac{S_T^{(2)}}{X_2} \right)^{-\xi_2} \right]. \tag{4.41}
\]

In consideration of \( S_T^{(i)} = S_0^{(i)} e^{Y_i^i}, \quad i = 1, 2 \), finally

\[
\hat{f}_p(\xi_1, \xi_2) = e^{-rT}X_1 \left( \frac{S_0^{(1)}}{X_1} \right)^{-\xi_1 - 1} \left( \frac{S_0^{(2)}}{X_2} \right)^{-\xi_2} \frac{1}{\xi_1(\xi_1 - 1)\xi_2} \left[ e^{-1 \xi_1 - \xi_2 Y_T^{1}} e^{-\xi_2 Y_T^{2}} \right]. 
\]

**Proof (Call case).** The price of two-asset correlation call option is given by

\[
C_T^p(K_1, K_2) = e^{-rT}E^* \left[ (S_T^{(1)} - K_1)^+ \mathbb{I}_{\{S_T^{(2)} > K_2\}} \right], \tag{4.42}
\]

where we use the same notations as in (4.40).

The Laplace transform is

\[
\hat{f}_c(\xi_1, \xi_2) = e^{-rT}X_1 \left( \frac{S_0^{(1)}}{X_1} \right)^{-\xi_1 - 1} \left( \frac{S_0^{(2)}}{X_2} \right)^{-\xi_2} \frac{1}{\xi_1(\xi_1 - 1)\xi_2} \left[ e^{-1 \xi_1 - \xi_2 Y_T^{1}} e^{-\xi_2 Y_T^{2}} \right]. 
\]

where \( k_i = -\log \left( \frac{K_i}{X_i} \right), \quad i = 1, 2 \).

The Fubini theorem justifies a change of the integration order and we get

\[
\hat{f}_c(\xi_1, \xi_2) = e^{-rT}X_1 E^* \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\xi_1 k_1} \left( e^{k_1} - \frac{S_T^{(1)}}{X_1} \right) \mathbb{I}_{\{k_1 > S_T^{(1)}/X_1\}} \right] dk_1 dk_2 
\]

\[
= e^{-rT}X_1 E^* \left[ \int_{-\log(S_T^{(1)}/X_1)}^{\infty} e^{-\xi_1 k_1} \left( e^{k_1} - \frac{S_T^{(1)}}{X_1} \right) dk_1 \int_{-\log(S_T^{(2)}/X_2)}^{\infty} e^{-\xi_2 k_2} dk_2 \right] 
\]

\[
= e^{-rT}X_1 E^* \left[ \frac{1}{\xi_1(\xi_1 + 1)} \left( \frac{S_T^{(1)}}{X_1} \right)^{\xi_1 + 1} \frac{1}{\xi_2(\xi_2 + 1)} \left( \frac{S_T^{(2)}}{X_2} \right)^{\xi_2} \right]. \tag{4.43}
\]
4.7.1 Calculation of the risk neutral expectation

We will calculate $E^*[e^{-\xi_1 Y_1} e^{-\xi_2 Y_2}]$. Consider the Lévy-Khintchine formula for the characteristic function,

$$E[e^{\theta Y}] = e^{T \varphi(\theta)}, \quad Y_T = (Y^{(1)}_T, Y^{(2)}_T),$$  \hspace{1cm} (4.44)

$$\varphi(\theta) = i(\theta, B) - \frac{1}{2}(\theta, C \theta) + \int_{\mathbb{R}^2} (e^{i(\theta, X)} - 1 - i(\theta, X) 1_{|X| \leq 1}) \nu(dx),$$  \hspace{1cm} (4.45)

where $\theta = (\theta_1, \theta_2)$,

$$B = \left(\begin{array}{cc} (r - \sigma_1^2/2), & (r - \sigma_2^2/2) \end{array}\right)^{tr},$$

$$C = \left(\begin{array}{cc} \sigma_1^2 & \sigma_1 \sigma_2 \rho \\ \sigma_2 \sigma_1 \rho & \sigma_2^2 \end{array}\right)$$

and in the case of no jumps

$$\nu(dx) = 0.$$  \hspace{1cm} (4.46)

Let put $i\theta_1 = -(\xi_1 - 1)$ and $i\theta_2 = -\xi_2$ in (4.45). Then

$$E^*[e^{-(\xi_1 - 1)Y^{(1)}_T} e^{-\xi_2 Y^{(2)}_T}] = \exp \left\{ \left( -(\xi_1 - 1)(r - \sigma_1^2/2)T - \xi_2(r - \sigma_2^2/2)T \right) 
+ \frac{1}{2} \left( -\frac{\sigma_1^2}{\sigma_2 \sigma_1 \rho} \right) \left( \begin{array}{cc} \sigma_1^2 & \sigma_1 \sigma_2 \rho \\ \sigma_2 \sigma_1 \rho & \sigma_2^2 \end{array}\right) \left( \begin{array}{c} -\xi_1 - 1 \\ -\xi_2 \end{array}\right) \right\}$$

$$\exp \left\{ T \left( - (\xi_1 - 1)(r - \sigma_1^2/2) - \xi_2(r - \sigma_2^2/2) 
+ \frac{1}{2} (\sigma_1^2 (\xi_1 - 1)^2 + 2\sigma_1 \sigma_2 \rho (\xi_1 - 1)\xi_2 + \sigma_2^2 \xi_2^2) \right) \right\}$$  \hspace{1cm} (4.46)

4.7.2 The moment generating function of the jump diffusion with independent jumps.

Again using formula (4.45) with $Y^{(i)}_T$ defined as in (4.39), we get

$$B = \left(\begin{array}{cc} (r - \sigma_1^2/2) + \lambda_1 p_1 \left( \frac{1 - e^{-\eta_1^1}}{\eta_1^1} - e^{-\eta_1^1} \right) - \lambda_1 q_1 \left( \frac{1 - e^{-\eta_1^2}}{\eta_1^2} - e^{-\eta_1^2} \right), \right.$$  \hspace{1cm} (4.47)

$$\left( r - \sigma_2^2/2 + \lambda_2 p_2 \left( \frac{1 - e^{-\eta_2^1}}{\eta_2^1} - e^{-\eta_2^1} \right) - \lambda_2 q_2 \left( \frac{1 - e^{-\eta_2^2}}{\eta_2^2} - e^{-\eta_2^2} \right) \right)^{tr}$$

$$C = \left(\begin{array}{cc} \sigma_1^2 & \sigma_1 \sigma_2 \rho \\ \sigma_2 \sigma_1 \rho & \sigma_2^2 \end{array}\right) \hspace{1cm} (4.47)$$

$$\theta = (- (\xi_1 - 1), - \xi_2)$$  \hspace{1cm} (4.48)
and \( \nu(dv) = \nu_1(dv_1)\nu_2(dv_2) \), where

\[
\nu_i(dv_i) = \lambda_i f_{V_i}(v_i)dv_i =
\lambda_i p_i \eta_1^i e^{-\eta_1^i v_1} \mathbb{I}_{[v_1 \geq 0]}dv_i + \lambda_i q_i \eta_2^i e^{\eta_2^i v_1} \mathbb{I}_{[v_1 < 0]}dv_i, \quad i = 1, 2 \tag{4.49}
\]

Put \( \lambda_i p_i \left( \frac{1 - e^{-\eta_1^i}}{\eta_1^i} - e^{-\eta_1^i} \right) - \lambda_i q_i \left( \frac{1 - e^{-\eta_2^i}}{\eta_2^i} - e^{-\eta_2^i} \right) = E[J_i], \quad i = 1, 2 \)

Then by (4.46) and (4.47), we get

\[
E^\star[\cdot] = \exp \left\{ T(-\xi_1 - 1)(r - \sigma_2^2/\eta_1^2) - \xi_2(r - \sigma_2^2/\eta_2^2) - (\xi_1 - 1)E[J_1] - \xi_2 E[J_2] + \frac{1}{2}(\lambda_1^2(\xi_1 - 1)^2 + 2\sigma_1\sigma_2\rho(\xi_1 - 1)\xi_2 + \sigma_2^2\xi_2^2) + I_1 \times I_2 \right\} \tag{4.50}
\]

Because the jumps are independent

\[
I_1 \times I_2 = \int_{-\infty}^{\infty} \left( e^{-(\xi_1 - 1)v_1} - 1 + (\xi_1 - 1)v_1 \mathbb{I}_{[|v_1| \leq 1]} \right) \lambda_1 f(v_1)dv_1 \times \int_{-\infty}^{\infty} \left( e^{-\xi_2 v_2} - 1 + \xi_2 v_2 \mathbb{I}_{[|v_2| \leq 1]} \right) \lambda_2 f(v_2)dv_2 \tag{4.51}
\]

for \( u = (-\xi_1 - 1, -\xi_2) \). Then each integral \( I_i \) is calculated according to

\[
\mathbf{I} = \int_0^\infty \left( e^{uv} - 1 - uv \mathbb{I}_{[|v| \leq 1]} \right) \lambda p_m e^{-m v}dv + \int_{-\infty}^0 \left( e^{uv} - 1 - uv \mathbb{I}_{[|v| \leq 1]} \right) \lambda q_m e^{uv}dv =
\lambda p_m \int_0^\infty e^{-(m-u)v}dv - \lambda p_m \int_0^\infty e^{-m v}dv - \lambda p_m \int_0^1 uv e^{-m v}dv + \lambda q_m \int_{-\infty}^0 e^{(m+u)v}dv - \lambda q_m \int_{-\infty}^0 e^{2 v}dv - \lambda q_m \int_0^1 uv e^{2 v}dv =
\lambda p_m \frac{1}{\eta_1 - u} - \lambda p_m \frac{1}{\eta_1} - \lambda p_m \frac{1}{\eta_1} \left( \frac{1}{\eta_1^2} - \frac{e^{-\eta_1}}{\eta_1} (\frac{1}{\eta_1} + 1) \right) + \lambda q_m \frac{1}{\eta_2 + u} - \lambda q_m \frac{1}{\eta_2} + \lambda q_m \frac{1}{\eta_2} \left( \frac{1}{\eta_2^2} - \frac{e^{-\eta_2}}{\eta_2} (\frac{1}{\eta_2} + 1) \right) =
\lambda p \frac{\eta_1}{\eta_1 - u} - \lambda p \frac{1}{\eta_1} \left( \frac{1}{\eta_1^2} - \frac{e^{-\eta_1}}{\eta_1} (\frac{1}{\eta_1} + 1) \right) + \lambda q \frac{\eta_2}{\eta_2 + u} - \lambda q + \lambda q \frac{1}{\eta_2} \left( \frac{1}{\eta_2^2} - \frac{e^{-\eta_2}}{\eta_2} (\frac{1}{\eta_2} + 1) \right) \tag{4.52}
\]
Chapter 5

Implementation of the method in a software package R

5.1 An application of the Kou model to the real market

Here we examine the solution of the real option pricing problem by using the the double exponential jump-diffusion model. The NASDAQ OMX Stockholm Market is considered, specifically the main index OMX Stockholm 30 (OMXS30). The OMXS30 is the market-weighted index, which consist of the 30 most actively traded stocks on the Stockholm Stock Exchange, such as Ericsson, Sandvik, Nokia, Scania and Volvo, Swedbank, SEB, Svenska Handelsbanken (SHB), Telia and Telia, Hennes & Mauritz etc.

The standardized option contracts in Sweden were introduced by a futures exchange OM AB (Optionsmaklarna) originated by Olof Stenhammar in the 1980s. OM acquired the Stockholm Stock Exchange in 1998 and merged with the Helsinki Stock Exchange (HEX) in September 2003. The join company got the name OMX. In the next few years OMX acquired the Copenhagen Stock Exchange, the Iceland Stock Exchange, Armenian Stock Exchange and Central Depository and took a part in the ownership of the Oslo Stock Exchange, launched alternative exchange for smaller companies in Denmark, Sweden, Iceland, Finland, started a virtual Nordic Stock Exchange. NASDAQ OMX Group was formed on May 2007, when OMX was bought by NASDAQ. The NASDAQ OMX Stockholm Market is is the part of the NASDAQ OMX Group Incorporated, which is owner of the American stock exchange NASDAQ Stock Market and seven European stock exchanges and currently is the world largest exchange company.

We observed the stocks of the Svenska Handelsbanken. The history refers
the foundation of Handelsbanken to 1871, when a number of prominent companies and individuals in Stockholm’s business world founded Stockholms Handelsbank. This was the result of a personal conflict at Stockholms Enskilda Bank which culminated in April 1871 with the resignation of eight board members who shortly after decided to form a new bank. Then it expands to become Svenska Handelsbanken.

Today it is one of the leading banks in the Nordic region and Europe’s most cost-effective bank. Handelsbanken has more than 460 branches in Sweden, and some branches in another counties, for example, in Denmark, Finland, USA, Poland, Russia, Norway, Austria, China, France and many others, see the official website of the bank (http://www.handelsbanken.com/). The bank also has an almost nationwide branch network in the other Nordic countries.

In Great Britain, Handelsbanken now has 50 branches. Handelsbanken aspires to be a universal bank, that is to take up the majority of the banking area, and in the last 2 decades, Handelsbanken has been widening its universal banking operations into not only different Nordic countries also in Great Britain. The Group consists of Handelsbanken Fonder (mutual funds), Handelsbanken Finans (finance company), Handelsbanken Liv (life insurance), Stadshypotek (mortgage company).

We also use in our observations the stock prices of swedish company Ericsson, the famous around the world provider of telecommunications equipment, related to mobile and fixed network operators. It was founded by Lars Magnus Ericsson in 1876, as a telegraph equipment repair shop. But nowadays it has customers in more than 175 countries, more than forty percent of the world mobile traffics are made through Ericsson system, it has 27 000 patents, they have already been for 134 years in the telecoms market. The head department located in Stockholm and company has offices in more than 175 countries, the number of workers just in Sweden is more than 17 000 people. This company is one of the largest and profitable in Sweden and it’s shares are widely traded in the markets.

We collected the data from the electronic information system SIX Edge™. The data from 24.04.2011-12.05.2011 with the frequency 1 minute, 5 minutes, 10 minutes, 20 minutes, 30 minutes, 1 hour and daily is at our disposal. There is 252 trading days in 2011.

5.2 The programme of the option valuation

In order to examine the correctness of the the Kou model, we decided to explore the theoretical issues of the model and consider the Laplace transform as the method of the pricing exotic options. There are programmes which
are written for a purpose to investigate the ability of the double exponential jump diffusion model to find the price of the plain vanilla option, but there is no available code concerning exotic options.

The aim of the practical part of the thesis was to write our own programme for pricing two asset correlated option. While the Laplace transform is presented as the option pricing method, the two-dimensional Euler algorithm is used for the the Laplace transform inverting. The main idea of the programme is to exploit in the practice the formulas (4.37), (4.38),(4.27). The programme is written in the statistical programming language $R$, which is free and available with description and manuals on the (http://www.r-project.org). We used version 2.13.0, which is described in [60]. However, for applying our program we need to find the parameters of the model $\lambda$, $\eta_1$, $\eta_2$, $p$.

The estimating of the parameters is beyond the scope of our thesis, it is quite broad problem and can be exploited as a theme for another research. We will just shortly discuss some of the methods to determine parameters. There are several of them from the effortless to the quite complicated. Any way in order to compute the parameters it is crucial to study logarithmic return process $Y_t = \ln \left( \frac{S_t}{S_{t-1}} \right)$.

The simplest way to find the parameters is based on the assumption that positive jumps occurs when the $Y_t$ exceed the standard deviation of returns on the certain amount, for examples three times as much, and negative jumps appears, when $Y_t$ is less then standard deviation on the certain amount. Then the amount of jumps during the time interval can be found. The intensity $\lambda$ is the ratio of the amount of jumps to amount of observation multiplied by the amount of the of the observation during the day. The probability of the upward jumps is ratio of the number of positive jumps to the total number of the jumps, $\eta_1$ and $\eta_2$ are the reverse of the arithmetic average of positive and negative jumps respectively. This method is not laborious, but there is shortcoming as you can not detect does jump occurs or it is just rapid change of the price.

The more complicated methods are based on the assumption, that the data set follows the considered model. Then parameters are estimated numerically correspondingly to the chosen method. There are generalized moment of methods, see [2], the simulated moment estimation, see [3], but we chose the Maximum Likelihood Estimation model(MLE), due to its suitable statistical properties. The consistent property of the MLE provides more precise results in the case of the high frequency data. We mostly used 20 minutes data which ensure the good precision and makes MLE preferable for us. We exploit the programme made by our classmate Wojiech Reducha and presented in his thesis, within of which he used Simulated Annealing numerical method for
maximizing the likelihood function, for more details see [51].

5.3 Results from the Kou model versus the Black-Scholes model

In this section we compare our results of the Kou model versus Black-Scholes model results. For these experiments we chose three different combinations of two underlying assets: the two-asset correlation option prices with OMXS30 and Ericsson B, the option prices with OMXS30 and SHB A and the option prices with Ericsson B and SHB A. There are three different starting point for options on 20.04.2011, 25.03.2011, 24.02.2011 with the same maturity date 12.05.2011, it means the durations are 23, 49 and 78 days. Here we present the results for the longest expiration date 24.02.2011-12.05.2011. The used parameters $\sigma$, $\lambda$ and $\eta$ for the Kou model are taken from the programme made by Wojciech Reducha. As interest rate we take STIBOR 3M on the yearly basis. The price $S(0)$ for each is the closing price from the 12.05.2011. For the Black-Scholes model we use volatility calculated by SIX Edge\textsuperscript{TM}.

The tables with results you can find in Appendix D and here we demonstrate the graphs of option prices with respect to different strike prices.

For this case on Figure 5.1 we chose as a first underlying asset Ericsson B with $S_1 = 95$, $\lambda_1 = 1, 7$, $\eta_{(1,1)} = 89$ and $\eta_{(1,2)} = 95$, $\sigma_1 = 0, 04$; then second underlying asset index OMXS30 with $S_2 = 1167$, $\lambda_2 = 1, 5$, $\eta_{(2,1)} = 72$ and $\eta_{(2,2)} = 78$, $\sigma_2 = 0, 015$; and the time to maturity $T = 0, 22$, interest rate $r = 0, 0246$ and the correlation $\rho = 0, 5$.

For the case on the Figure 5.2 we chose as a first underlying asset Ericsson B with $S_1 = 95$ and $\sigma_1 = 0, 21$, then second underlying asset index OMXS30 with $S_2 = 1167$ and $\sigma_2 = 0, 15$; and the time to maturity $T = 0, 22$, interest rate $r = 0, 0246$ and the correlation $\rho = 0, 5$.

The discrepancy between prices calculated under the considered models is not significant. However, we can see that the price of the two asset correlation call option is more sensitive to the high values of the Ericsson B strike price in the case of the Kou model. It explains that the shape of plot which is correspondent to the jump diffusion model is more figured.

For the case on the Figure 5.3 we chose as a first underlying asset SHB A with $S_1 = 210$, $\lambda_1 = 1, 4$, $\eta_{(1,1)} = 80$ and $\eta_{(1,2)} = 103$, $\sigma_1 = 0, 019$; then second underlying asset index OMXS30 with $S_2 = 1167$, $\lambda_2 = 1, 5$, $\eta_{(2,1)} = 72$ and $\eta_{(2,2)} = 78$, $\sigma_2 = 0, 015$; and the time to maturity $T = 0, 22$, interest rate $r = 0, 0246$ and the correlation $\rho = 0, 41$. 
For the case on the Figure 5.1, we chose as a first underlying asset SHB A with $S_1 = 210$ and $\sigma_1 = 0.16$, then second underlying asset index OMXS30 with $S_2 = 1167$ and $\sigma_2 = 0.15$; and the time to maturity $T = 0.22$, interest rate $r = 0.0246$ and the correlation $\rho = 0.41$.

We can detect the similar observations as in the previous case. The plot correspondent to the BSM model is more flat and less sensitive to the increase of the strike price $K_2$.

For the case on the Figure 5.2, we chose as a first underlying asset Ericsson B with $S_1 = 95$, $\lambda_1 = 1.7$, $\eta_{(1,1)} = 89$ and $\eta_{(1,2)} = 95$, $\sigma_1 = 0.04$; then second underlying asset SHB A with $S_2 = 210$, $\lambda_2 = 1.4$, $\eta_{(2,1)} = 80$ and $\eta_{(2,2)} = 103$, $\sigma_2 = 0.019$; and the time to maturity $T = 0.22$, interest rate $r = 0.0246$ and the correlation $\rho = -0.8$.

For the case on the Figure 5.3, we chose as a first underlying asset Ericsson B with $S_1 = 95$, $\lambda_1 = 1.7$, $\eta_{(1,1)} = 89$ and $\eta_{(1,2)} = 95$, $\sigma_1 = 0.04$; then second underlying asset SHB A with $S_2 = 210$, $\lambda_2 = 1.4$, $\eta_{(2,1)} = 80$ and $\eta_{(2,2)} = 103$, $\sigma_2 = 0.019$; and the time to maturity $T = 0.22$, interest rate $r = 0.0246$ and the correlation $\rho = -0.8$. 
In the case of the two asset correlation call option on the Ericsson B and SHB stocks difference between shapes of the graphs is more significant than in previous cases.

And now for more detailed analysis we present the results, which show the dependence of two-asset correlation call option prices with underlying assets OMXS30 and SHB A on changing of correlation between them. For the experiment we use the same values as for the Figures 5.3 and 5.4. Hence we can say that the option prices and correlation between two underlying assets are linear dependent. The price of option rises with increase of correlation in the both models.
Figure 5.3: Results of Kou model for the two-asset correlation options with underlying assets OMXS30 and SHB A.

Figure 5.4: Results of Black-Scholes model for the two-asset correlation options with underlying assets OMXS30 and SHB A.
Figure 5.5: Results of Kou model for the two-asset correlation options with underlying assets Ericsson B and SHB A.

Figure 5.6: Results of Black-Scholes model for the two-asset correlation options with underlying assets Ericsson B and SHB A.
Figure 5.7: The dependence of the option price on the change of correlation in the case of jump-diffusion and diffusion models. Prices of the two-asset correlation call options with underlying assets OMXS 30 and SHB A.
CHAPTER 5. IMPLEMENTATION OF THE METHOD IN A SOFTWARE PACKAGE R
Chapter 6

Conclusions

The aim of our work was to explore theoretical framework and assumptions of the double exponential jump diffusion model and to assess the Laplace transform as option pricing method under the given model. The motivation of the choice of the Kou model is that the model possess crucial features, since it reproduces leptokurtic and asymmetric features and the "volatility smile" effect, which can be useful for the option pricing. Moreover, the Kou model is tractable and can be easily interpreted economically and it is feasible to obtain the closed form solutions for various option pricing problems, such as exotic options, under the Kou model, while it is quite difficult using many other models. This feature is the decisive factor of the model choice, due to the fact that exotic options are becoming more popular among the market participants. The theoretical knowledge concerning to the two-dimensional Euler algorithm of the Laplace transform inversion was used to write the program for the pricing two asset correlation option, relatively modern type of the options.

The price of the option obtained by the programme under the Kou model is compared with one, which is provided by available option pricing calculator, based on the Black-Scholes model. In most cases the difference is not significant. The discrepancy becomes more tangible in case of the two asset correlation call option written on the Ericsson B and SHB A underlying assets. The reason of the difference is a broad theme. The detection of the reasons may serve as the aim of the many researches on the considered field, as it is not feasible to find reasons based only on our works and available papers of other authors. We may assume that the significant difference in case of the the Ericsson B and SHB A underlying assets is caused by the fact that the average volatility of these stocks is the biggest among all considered combinations. Also the reason can be that the correlation between the returns of the Ericsson B and SHB A stocks is low and negative. However in order to
make such statement we need more researches. Unfortunately, the two asset correlation option is not on the electronic exchange and we can not compare our results with real prices and we do not have enough information to draw a conclusion about performance of the Kou model on NASDAQ OMX Stockholm Market. We can not assert that the Kou model perform better on the considered market, than the the Black-Scholes model. Nevertheless, we can state that the double exponential jump-diffusion model implies that the price of exotic options can be calculated numerically by using Laplace transform, while the Euler algorithm is the fast and accurate method of the Laplace transform inversion. The considered option pricing method can be implemented to the real market and it will be useful for the derivative market participants. Despite to the impossibility to check performance of the Kou model on the two asset correlation option pricing, it is evident that the trader will have better position possessing several option pricing methods the only one. Moreover, we concluded that our option pricing programme is correct and effective. We could check it by putting zero jumps, which leads to the coincidence of the results obtained by our programme and from available Black-Scholes calculator. Taking into account that the Kou model is the modification of the Black-Scholes model with assumption of the existence of the jumps, the coincidence is theoretically explicable. In addition the programme is universal, as it can provide results under the both models. We recommend for the further researches to examine the the considered method for other types of the exotic options, to implement the Laplace transform on the Merton model and to investigate some other models, which can incorporate the volatility clustering effect, such as affine jump-diffusion models, since it is combination of the jump diffusion and stochastic volatility models. Probably, these type of model are more suitable for the pricing of the long term options or options on the highly volatile assets then the Kou model.
**Notation**

\( \mathbb{R} \) real numbers.

\( \mathbb{R}^2 \) or \( \mathbb{R}^+ \times \mathbb{R} \) a two-dimensional space of pairs of real numbers - the real plane. A point or vector in \( \mathbb{R}^2 \) has two coordinates.

\( X = (X_t)_{t \geq 1} \) the stochastic process or the sequence of random variables, which depend on \( t \) and \( t \geq 1 \).

\( (\Omega, \mathcal{F}, (\mathcal{F}_n)_{n \geq 1}, P) \) filtered probability space with set of outcomes \( \Omega \), sigma algebra \( \mathcal{F} \), flow or filtration \( \{\mathcal{F}_n\} \) and probability measure \( P \).

\( E^*(X) \) expectation of the random variable \( X \) with respect to the risk neutral measure.

\( 1_{\{|y| \leq 1\}} \) the indicator function of event that \( |y| \leq 1 \).

\( \rho(S_1, S_2) \) the correlation between two different underlying assets.

\( S(t^-) \) the asset price before jump.

\( Y_i \) and \( \{i = 1, 2, ..., N(x)\} \) process \( Y_i \) is normal distributed.

\( Re(\hat{f}(\xi)) \) the real part of complex function.

\( Im(\hat{f}(\xi)) \) the imaginary part of complex function.

\( S_n^{(i)}, Y_T^{(i)} \) respectively the price and the return process of asset \( i \), \( i = 1, 2 \).

\( C_T \) the price of two-asset correlation call option.

\( P_T \) the price of two-asset correlation put option.

\( \hat{f}_C(\xi_1, \xi_2) \) the Laplace transform of the two-asset correlation call option price.

\( \hat{f}_P(\xi_1, \xi_2) \) the Laplace transform of the two-asset correlation put option price.
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Appendix

APPENDIX A: Direction calculations of the risk-neutral expectation by solving double integral for two-asset correlation put option under Black-Scholes model

Let $S_1$ and $S_2$ be the asset prices for some correlated stocks and $K_1, K_2$ the respective strike prices. Then the price of a two-asset put correlation option is given by

$$ P(K_1, K_2) = e^{-rT} E^*[ (K_1 - S_1)_+ 1\{S_2, T < K_2\}], $$

(6.1)

where $r$ is the interest rate, $T$ is time to maturity, $E^*$ is the risk-neutral expectation.

The two stocks are assumed to follow a geometric Brownian motion

$$ S_t^{(i)} = S_0^{(i)} e^{(r - \sigma_i^2/2)t + \sigma_i B_t^{(i)}}, \quad i = 1, 2, $$

(6.2)

and

$$ \rho = \text{cor}(B_t^{(1)}, B_t^{(2)}). $$

(6.3)

The Laplace transform of $P(K_1, K_2)$ is

$$ \mathcal{L}(\xi_1, \xi_2) = e^{-rT} X_1 \left( \frac{S_1}{X_1} \right)^{-\xi_1} \left( \frac{S_2}{X_2} \right)^{-\xi_2} \frac{1}{\xi_1(\xi_1 - 1)\xi_2} E^* \left[ e^{-(\xi_1 - 1) Y_T^{(1)}} e^{-\xi_2 Y_T^{(2)}} \right], $$

(6.4)

where

$$ Y_T^{(i)} = (r - \sigma_i^2/2)T + \sigma_i B_T^{(i)}, \quad i = 1, 2, $$

are the processes of the log returns.

Calculation of the risk-neutral expectation,

$$ E^*[\cdot] = E^* \left[ e^{-(\xi_1 - 1) Y_T^{(1)}} e^{-\xi_2 Y_T^{(2)}} \right], $$

where

$$ Y_T^{(i)} = (r - \sigma_i^2/2)T + \sigma_i B_T^{(i)} = (r - \sigma_i^2/2)T + \sigma_i \sqrt{T} B_1^{(i)}, \quad i = 1, 2, $$

(6.5)
where

\[(B_1^{(1)}, B_1^{(2)}) \sim \mathcal{N} \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right) \]  

(6.6)

and \( \rho \) is the correlation coefficient.

\[E^*[\cdot] = e^{-(\xi_1 - 1)(r - \sigma_1^2/2)^T} e^{-(\xi_2 - \sigma_2^2/2)^T} E^*\left[e^{-\sigma_1 \sqrt{T}(\xi_1 - 1)B_1^{(1)}} e^{-\sigma_2 \sqrt{T} \xi_2 B_1^{(2)}} \right]. \]  

(6.7)

Put

\[I = E^*\left[e^{-\sigma_1 \sqrt{T}(\xi_1 - 1)B_1^{(1)}} e^{-\sigma_2 \sqrt{T} \xi_2 B_1^{(2)}} \right], \]

(6.8)

\[u = \sigma_1 \sqrt{T}(\xi_1 - 1)2(1 - \rho^2), \]

\[\nu = \sigma_2 \sqrt{T} \xi_2 2(1 - \rho^2). \]

Then

\[I = \frac{1}{2\pi \sqrt{1 - \rho^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \frac{1}{(1 - \rho^2)} \{x^2 + ux - 2\rho xy + y^2 + \nu y\}} \, dx \, dy, \]

\[I = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \frac{1}{(1 - \rho^2)} \{x^2 + ux\}} \left\{ \frac{1}{\sqrt{2\pi \sqrt{1 - \rho^2}}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \frac{1}{(1 - \rho^2)} \{y^2 + (\nu - 2\rho x) y\}} \, dy \right\} \, dx \]

\[= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \frac{1}{(1 - \rho^2)} \{x^2 + ux\}} I_x \, dx, \]

where

\[I_x = \frac{1}{\sqrt{2\pi \sqrt{1 - \rho^2}}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \frac{1}{(1 - \rho^2)} \{y^2 + (\nu - 2\rho x) y + \frac{(\nu - 2\rho x)^2}{4} - \frac{(\nu - 2\rho x)^2}{4}\}} \, dy, \]

\[I_x = \frac{1}{\sqrt{2\pi \sqrt{1 - \rho^2}}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \frac{1}{(1 - \rho^2)} \{y + \frac{(\nu - 2\rho x)^2}{2}\}} e^{\frac{(\nu - 2\rho x)^2}{2(1 - \rho^2)}} \, dy = e^{\frac{(\nu - 2\rho x)^2}{2(1 - \rho^2)}}, \]

we then get
\[
I = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left(1 - \rho^2\right) \left(x^2 + ux - \frac{(u+\nu)^2}{4}\right)} \, dx \\
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left(1 - \rho^2\right) \left(x^2 + x(u+\nu) - \frac{(u+\nu)^2}{4}\right)} \, dx \\
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left(x^2 + \frac{2x(u+\nu)}{2(1-\rho^2)} - \frac{x^2}{4(1-\rho^2)}\right)} \, dx \\
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left(x^2 + \frac{(u+\nu)^2}{2(1-\rho^2)} - \frac{x^2}{4(1-\rho^2)}\right)} \, dx \\
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left((x-\frac{(u+\nu)^2}{2(1-\rho^2)})^2 + \frac{x^2}{4(1-\rho^2)}\right)} \, dx \\
= e^{-\frac{u^2 + 2u\nu + \nu^2}{8(1-\rho^2)^2}}.
\]

Finally, the risk-neutral expectation equal
\[
E^*\left[e^{-\xi_1 Y_T^{(1)}} e^{-\xi_2 Y_T^{(2)}}\right] = e^{-\left(\xi_1 - 1\right) \left(r - \sigma_1^2 / 2\right) T} e^{-\xi_2 \left(r - \sigma_2^2 / 2\right) T} e^{-\frac{u^2 + 2u\nu + \nu^2}{8(1-\rho^2)^2}},
\]

where
\[
u = \sigma_2 \sqrt{T} \xi_2 \sigma_2 \left(1 - \rho^2\right),
\]
\[
u^2 \left(1 - \rho^2\right) = \frac{\sigma_2^2}{\sigma_1^2} \left(1 - \rho^2\right).
\]

\[
E^*\left[e^{-\xi_1 Y_T^{(1)}} e^{-\xi_2 Y_T^{(2)}}\right] = e^{-\left(\xi_1 - 1\right) \left(r - \sigma_1^2 / 2\right) T - \xi_2 \left(r - \sigma_2^2 / 2\right) T} e^{\frac{1}{2} \left(\sigma_1^2 (\xi_1 - 1)^2 + 2\sigma_1 \sigma_2 \rho (\xi_1 - 1) \xi_2 + \sigma_2^2 \xi_2^2\right)}.
\]
APPENDIX B:
Program code for pricing two-asset correlation call options under Kou model via the Laplace transform.

# Two asset correlation call option pricing by using
# the Laplace transform
#
#
#
#
#
Bcall<-function(S1,S2,K1,K2,r,T,sigma1,sigma2,rho,l1,l2) {

A<-40;
theta1<-2/(sigma1*sqrt(T));
theta2<-2/(sigma2*sqrt(T));

k1<-min(A/theta1,A/4);
k2<-min(A/theta2,A/4);

mu1<-(r-sigma1^2/2)*T;
mu2<-(r-sigma2^2/2)*T;

X1<-S1*exp(k1);
X2<-S2*exp(k2);
N<-200;

################################################################################ Double exponential parameters ################################################################################
###l1<-1.5;
###l2<-1.7;
etal1<-71.6
eta12<-78.3
eta21<-88.7
eta22<-95.0
p1<-0.5
p2<-0.5
q1<-1-p1

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q2<-1-p2

####### The Laplace transform of the price function #######

uic<-function(s1,s2){

fp<-exp(-r*T)*X1*(S1/X1)^(1+s1)*(S2/X2)^(s2)/(s1*
(s1+1)*s2)*exp(mu1*(s1+1)+mu2*s2)* # mean diffusion
exp(1/2*(sigma1^2*(s1+1)^2+2*sigma1*sigma2*
(s1+1)*s2*rho+sigma2^2*s2^2)*T)* # expectation of diffusion
exp((s1+1)*(l1*p1*((1-exp(-eta11))/eta11-
exp(-eta11)))-l1*q1*((1-exp(-eta12))/eta12-
exp(-eta12)))*T) # mean S1 jumps
exp(s2*(l2*p2*((1-exp(-eta21))/eta21-
exp(-eta21)))-l2*q2*((1-exp(-eta22))/eta22-
exp(-eta22)))*T) # mean S2 jumps

exp(T*(l1*p1*eta11/(eta11-(s1+1))-
11-l1*p1*eta11*(s1+1)*(1/eta11^2-
exp(-eta11)/eta11^2-exp(-eta11)/eta11)+
11*q1*eta12/(eta12+(s1+1))+11*q1*eta12*(s1+1)
*(1/eta12^2-exp(-eta12)/eta12^2-
exp(-eta12)/(eta12)))*l2*p2*eta21/((eta21-s2)-
l2-l2*p2*eta21*s2)*l2*eta21^2-exp(-eta21)/eta21^2-
exp(-eta21)/eta21)+l2*eta22*eta22/((eta22+s2)+
l2*eta22^2*(1/eta22^2-exp(-eta22)/eta22^2-
exp(-eta22)/eta22))

return(fp)
}

####### The two-dimensional Laplace inversion #######

tvaDpet<-function(lpfun,t1,t2){

A1<-40; # A1 and A2 are predefined, see Petrella 2004
A2<-18.4;
N<-400;
M<-N;
a1<-A1/2;
a2<-A2/2;
D<- exp(a1+a2)/(4*t1*t2);
E<- sum((-1)^(0:N)*Re(exp(-pi*1i)*lpfun(a1/t1,a2/t2-pi/t2*1i-pi/t2*(0:N)*1i)));
Fsum<-0;
for(j in 0:M){
  Fsum<-Fsum+(-1)^j*sum((-1)^(0:N)*Re(exp(-pi*2i)*lpfun(a1/t1-pi*1i/t1-j*pi*1i/t1,a2/t2-pi/t2*1i-pi/t2*(0:N)*1i)));
  G<- sum((-1)^(0:M)*Re(exp(-pi*1i)*lpfun(a1/t1-pi*1i/t1-(0:M)*pi*1i/t1,a2/t2)));
  Hsum<- 0;
  for(j in 0:M){
    Hsum<-Hsum+(-1)^j*sum((-1)^(0:N)*Re(1*lpfun(a1/t1-pi*1i/t1-j*pi*1i/t1,a2/t2+pi*1i/t2+(0:N)*pi*1i/t2)));
  }
return(D*(lpfun(a1/t1,a2/t2)+2*E+2*Fsum+2*G+2*Hsum))
}
tvaDpet(uic,-log(K1/X1),-log(K2/X2));

# Example of usage:
# Bcall(S1,S2,K1,K2,r,T,sigma1,sigma2,rho,l1,l2)
# Bcall(100,105,105,115,0.05,1,0.3,0.2,0.5,1.4,1.7)
APPENDIX C:
Program code for pricing two-asset correlation put options under Kou model via the Laplace transform.

```r
# Two assert correlation put option pricing by using the Laplace transform

Bcall<-function(S1,S2,K1,K2,r,T,sigma1,sigma2,rho) {

  A<-40;
  theta1<-2/(sigma1*sqrt(T));
  theta2<-2/(sigma2*sqrt(T));

  k1<-min(A/theta1,A/4);
  k2<-min(A/theta2,A/4);

  mu1<-(r-sigma1^2/2)*T;
  mu2<-(r-sigma2^2/2)*T;

  X1<-S1*exp(-k1);
  X2<-S2*exp(-k2);
  N<-200;
  # Double exponential parameters
  l1<-0;
  l2<-0;
  eta11<-30
  eta12<-30
  eta21<-30
  eta22<-30
  p1<-0.5
  p2<-0.5
  q1<-1-p1
```

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q2<-1-p2

########## The Laplace transform of the price function ##########

uic<-function(s1,s2){

fp<-exp(-r*T)*X1*(S1/X1)^(1-s1)*(S2/X2)^(-s2)/
(s1*(s1-1)*s2)*exp(-mu1*(s1-1)-mu2*s2)* # mean diffusion
exp(1/2*(sigma1^2*(s1-1)^2+2*sigma1*sigma2*rho+
(s1-1)*s2*sigma2^2*s2^2)*T)* # expectation of diffusion
exp(l1*p1*((1-exp(-eta11))/eta11-
exp(-eta11))-l1*q1*(((1-exp(-eta12))/eta12-
exp(-eta12)))*T)* # mean S1 jumps
exp(l2*p2*((1-exp(-eta22))/eta22-
exp(-eta22)))*T)* # mean S2 jumps
exp(l1*p1+eta11/eta11+(s1-1)-l1-
11*p1+eta11*(s1-1)*(1/eta11)^2-exp(-eta11)/eta11-
exp(-eta11)+l1*q1+eta12/eta12-(s1-1)-
l1*q1+eta12*(s1-1)*(1/eta12)^2-exp(-eta12)/eta12-
exp(-eta12)+l2*p2+eta21/eta21+(s2)-
l2+12*p2*eta21*(s2)*1/eta21^2-exp(-eta21)/eta21-
exp(-eta21)+l2*q2+eta22/eta22-(s2)-
l2+12*q2*eta22*(s2)*1/eta22^2-exp(-eta22)/eta22-
exp(-eta22))

return(fp)
}

########## The two-dimensional Laplace inversion ##########

tvaDpet<-function(lpfun,t1,t2){

A1<-40; # A1 and A2 are predefined, see Petrella 2004
A2<-18.4;
N<-400;

M<-N;
a1<-A1/2;
a2<-A2/2;
D<- exp(a1+a2)/(4*t1*t2);
E<- sum((-1)^(0:N)*Re(exp(-pi*1i)*lpfun(a1/t1,a2/t2-pi/t2*1i-pi/t2*(0:N)*1i)));
Fsum<-0;
for(j in 0:M){
  Fsum<-Fsum+(-1)^j*sum((-1)^(0:N)*Re(exp(-pi*2i)*lpfun(a1/t1-pi*1i/t1-j*pi*1i/t1,a2/t2-pi/t2*1i-pi/t2*(0:N)*1i)));
G<- sum((-1)^(0:M)*Re(exp(-pi*1i)*lpfun(a1/t1-pi*1i/t1-(0:M)*pi*1i/t1,a2/t2)));
Hsum<- 0;
for(j in 0:M){
  Hsum<-Hsum+(-1)^j*sum((-1)^(0:N)*Re(exp(-pi*2i)*lpfun(a1/t1-pi*1i/t1-j*pi*1i/t1,a2/t2+pi*1i/t2+(0:N)*pi*1i/t2))));
return(D*(lpfun(a1/t1,a2/t2)+2*E+2*Fsum+2*G+2*Hsum))
}

tvaDpet(uic,log(K1/X1),log(K2/X2));

# Example of usage:
# Bcall(S1,S2,K1,K2,r,T,sigma1,sigma2,rho,l1,l2)
# Bcall(100,105,105,115,0.05,1,0.3,0.2,0.5,1.4,1.5)
Table 6.1: Results of pricing under the Kou model with different strike prices in case of two-asset correlation

Option SHB and OHIX 30. Part I

APPENDIX D:
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Table 6.3: Results of pricing under the Black-Scholes model with different strike prices in case of two-asset correlation.
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Table 6.5: Results of pricing under the Kou model with different strike prices in case of two-asset correlation.
Table 6.6: Results of pricing under the Kou model with different strike prices in case of two-asset correlation options (Ericsson B and OMXS30). Part II.

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Table 6.7: Results of pricing under the Black-Scholes model with different strike prices in case of two-asset correlation
Table 6.8: Results of pricing under the Black-Scholes model with different strike prices in case of two-asset correlation options (Ericsson B and OMXS30). Part II.

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