Correlation between Sector Indices of OMX Stockholm Exchange Market

Master’s Thesis in Financial Mathematics

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Correlation between Sector Indexes of OMX Stockholm Exchange Market

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Preface

I am very pleased to say thanks for all people who helped me to write this project. It obviously to mention my supervisor Jan Olof Johanson who helped me to choose the right theme and go in right direction. My respect and confession to group coordinator Ljudmila Bordag. Thanks a lot to all my classmates of the Financial Mathematics Program.
Abstract
In this paper we aim to investigate volatility and correlation of sector indexes of Nordic Market. More precisely we work with OMX Stockholm Exchange Indexes, considering the Paper, the Energy and the Bank sectors.
We use daily returns over the period from 5 January 2001 to 13 April 2007 and compute and forecast return volatility using the $GARCH(1,1)$ model. We also calculate the correlation matrix of the indexes.
The $GARCH(1,1)$ model fit the empirical data well for all three sectors and can therefore be used for volatility forecasts. Here, we have predicted the one-day-ahead forecasts and based on these data calculated the correlation matrix. The results from these calculations show that all three sectors are highly correlated. We obtained however the smallest correlation between Paper and Energy which was surprising as the Paper industry is very energy consuming. This result indicates other relations between Paper and Energy.
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Chapter 1

Introduction

Investors need to design portfolios based on different wishes for example sectors, stocks, options, production facilities or caps. Therefore a study of the correlation between various portfolio constituents based on market risk measurements is well motivated in recent time.

We decided to make our investigation in area of OMX, which possesses and operates the largest united market of securities in Northern Europe. OMX is a prominent provider of marketplace services and solutions to such a markets as financial and energy around the world. More precisely we are going to deal with Stockholm Exchange Market.

A new organization of OMX in October 2006 fastened changes in structure of market and therefore recasts in lists of companies and indexes. Due to this reorganization not so many investigations has been made, especially not in the field of sector indexes. Therefore researches of various structures of OMX should be essential.

In this work we determined to investigate relations between different sector indexes of OMX Stockholm Exchange. More precisely, we aim to analyze the correlation between Energy, Bank and Paper & Forest Sector Indexes. The particular choice of these indices is motivated by following. It is obvious and understandable that Energy and Paper industry are interrelated between each other. So Energy and Paper & Forest Indexes should be highly correlated. As for Bank Index, is not so obvious how it can be related with other. We explore correlation between these things by comparing their volatilities.

Volatility data of financial return is swayed by time dependent information flows which result in evident temporal volatility clustering. These time series can be parameterized using GARCH models, which was introduced by Bollerslev (1986). It has been determined that these models can provide fine in-sample estimates of parameter and, when the becoming volatility measure is used, trustworthy out-of-sample volatility forecasts. GARCH-type
models possibly due to their naivety and flexibility have since become very far-famed essentially in financial world. Forecasting of volatility in itself is useful instrument of assistance in decision-making for traders or different financial companies. Andersen and Bollerslev (1998), Bollerslev, Chou and Kroner (1992), Hol and Koopman (2000) in their investigation papers inspected the predictive precision of out-of sample volatility forecasts grounded on $GARCH$. Andersen, Bollerslev and Lange (1999) observed a considerable amelioration in the performance of out-of-sample predicting of the $GARCH$ model.

However, sector indexes often seem to exhibit a strong asymmetry, whereby a negative return boosts volatility by more than a positive return of the same absolute magnitude. Hol and Koopman (2002) showed that $GARCH$ model is readily generalized to capture this effect. Andersen and Bollerslev (2005), Andersen, Bollerslev, Christoffersen and Diebold (2005) in their works examined the question of $GARCH$ volatility and correlation.

We can determine our research question as investigating the forecasting volatility, based on $GARCH(1, 1)$ model, and consequently volatility correlation between sector indexes of OMX Stockholm Exchange Market.

In Chapter 2 we discuss theoretical background of our investigation. In the sections of this chapter we introduce existing volatility models; $GARCH(p, q)$ model and the case when $p = 1$ and $q = 1$. Also in this section we present methods and models for our exploration. All our investigations are included in Chapter 3. Here are computation results for index return data, forecasting volatility and correlation. Chapter 4 represents main conclusions.
Chapter 2

Theory and Methods

2.1 Volatility and Volatility models

The idea of volatility is essential to many applied issues in the world of finance and financial engineering. Using a more simple language, volatility backs to the fluctuations which are observed in certain event over period of time. Up to the recent time, we can find great variety of volatility models which in one’s turn are grounded on different parameters.

If we will look on volatility from the side of statistics, then we simply can represent it in following form

\[ \hat{\sigma} = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (r_t - \tau)^2}, \]

where \( T \) - number of days (period), \( t \) - observed day, \( \tau \) - the average in a period and \( r_t \) is daily returns.

2.1.1 Volatility concepts

According to numerous papers and works, we can classify the main volatility concepts in the following way. We mark main volatility types, which should be essential for our investigations. Let us consider time interval \([t - h, t]\), where \(0 < h \leq t \leq T\). We take logarithmic price \(p(t)\), defined on \((\Omega, \mathcal{F}, P)\). We define the return series as

\[ r(t, h) = p(t) - p(t - h) \] (2.1)
Chapter 2. Theory and Methods

1. actual (or notional) volatility
   Usually notional volatility is approximated by historical volatility. It is useful to mention that this type of volatility is appropriate only for the stationary time series. So, actual volatility over the period is
   \[
   \nu^2(t, h) \equiv [M_t, M_t] - [M, M]_{t-h} = [M^c_t, M^c_t] - [M^c, M^c]_{t-h} + \sum_{t-h<s\leq t} \Delta M^2(s),
   \]
   where \(M\) is a local martingale.
   The actual volatility equals the quadratic variation for \(r_t^2\). This volatility catches the sample trajectory variability of \(r_t\) over time period.

2. expected volatility
   This volatility concept defined by
   \[
   \varphi^2(t, h) = \mathbb{E} \left[ (r_t - \mathbb{E}(\mu(t, h)|\mathcal{F}_{t-h}))^2 | \mathcal{F}_{t-h} \right] = \mathbb{E} \left[ (r_t - m(t, h))^2 | \mathcal{F}_{t-h} \right],
   \]
   where \(m(t, h)\) - expected returns\(^3\). Especially, the future return inconstancy in equation above shows real return and intra-period innovations.

3. spot (or instantaneous) volatility
   This concept of volatility at time \(t\), \(0 \leq t \leq T\) is described by
   \[
   \sigma^2_t \equiv \lim_{h \to 0} \left[ \mathbb{E}(([M^c_t, M^c_t] - [M^c, M^c_{t-h}])/h) | \mathcal{F}_{t-h} \right],
   \]
   where \(\mathcal{F}_{t-h}\) is a filtration.

2.1.2 Discrete-time volatility models

Within this group of models, the most essential difference refers to the character of the variables in \(\mathcal{F}_{t-h}\). Discrete-time volatility models definitely parameterize the expected volatility, \(\varphi^2(t, h), h > 0\). The main types of volatility models in this class are:

\(^2\)See details in Andersen, Bollerslev and Diebold (2002)

\(^3\)According to Andersen, Bollerslev and Diebold (2002) \(r_t - m(t, h) = (\mu(t, h) - m(t, h)) + M(t, h)\).
1. **stochastic volatility model**
   In the terms of discrete time, simple stochastic volatility model with 
   \( \mathbb{E}(\varepsilon_t) = 0, \mathbb{E}(\eta_t) = 0, \text{Var}(\varepsilon_t) = 1 \) and \( \text{Var}(\eta_t) = \beta^2 \) can have following 
   representation

   \[
   r(t) = \sigma_t \varepsilon_t, \\
   \sigma_t^2 = \sigma^2 \exp(h_t) \\
   h_t = \phi h_{t-1} + \sigma_t \eta_t
   \]

   where \( \sigma^2(t) \) - instantaneous volatility; \( r(t) \) defined as in 2.1; \( h_t \) - stochastic 
   process; \( 0 < \phi < 1 \) - the parameter of persistence which guarantee 
   the stationarity of \( \sigma_t^2 \); \( \sigma_t^2 \) - scaling factor.

   We refer here to Hol and Koopman (2002), Ghysels, Harvey and Re-

2. **ARCH and GARCH models**
   As for these types of volatility modeling, we will explain later.

There exist also continuous-time volatility models, which concern spot volatil-

2.2 **GARCH\((p,q)\)** Model

Appeal to non-linear stochastic conditional Gaussian models is provoked by 
the desire and necessity to find the explanation to observed phenomenons 
in financial statistics and economics (clusterization, "fat" tails, existence of 
long memory in assets and asset prices).

According to Borellslev (1986) and Engle (1982) we can introduce and 
give the description of the **GARCH** - Generalized Autoregressive Conditional 
Heteroscedastic model.

Let \( (\Omega, \mathcal{F}, \mathbb{P}) \) - initial probability space, \( (\varepsilon_t)_{t\geq 1} \) - sequence of independent 
normally distributed random variables. Let us denote \( \mathcal{F}_n \) -\( \sigma \)-algebra of 
\( \sigma(\varepsilon_1, \ldots, \varepsilon_t) \), \( \mathcal{F}_0 = (\emptyset, \mathcal{F}_0) \).

Refers to Engle (1982), who goes to conditional Gaussian model, we write

\[
y_t = \sigma_t \varepsilon_t \quad \text{(2.2)}
\]

The **ARCH\((q)\)** process is given by following equations

\[
\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2), \\
\sigma_t^2 = \omega + \beta_1 \varepsilon_{t-1}^2 + \ldots + \beta_q \varepsilon_{t-q}^2.
\]

\({}^4\text{given function}\)
Ωₜ - history of the process εₜ, Ωₜ₋₁ - pre-history of it.

For deriving the GARCH model, we use infinite-dimensional geometrical lag in the ARCH model.

\[ \sigma_t^2 = \omega + \beta \sum_{j=1}^{\infty} \alpha^j \varepsilon_{t-j}^2 = \omega + \frac{\beta}{1 - \alpha L} \varepsilon_{t-1}^2, \]  

(2.3)

where \( L \) is a backshift operator. We rewrite the formula 2.3 as

\[ \sigma_t^2 = (1 - \sigma) \omega + \alpha \sigma_{t-1}^2 + \beta \varepsilon_{t-1}^2. \]

GARCH(\( p, q \)) model is generalized expression above, and we get

\[ \sigma_t^2 = \omega + \alpha_1 \sigma_{t-1}^2 + \ldots + \alpha_p \sigma_{t-p}^2 + \beta_1 \varepsilon_{t-1}^2 + \ldots + \beta_q \varepsilon_{t-q}^2. \]

Consequently, GARCH(\( p, q \)) process is given by

\[ \varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2), \]  

(2.4)

\[ \sigma_t^2 = \omega + \sum_{j=1}^{p} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2. \]  

(2.5)

In this case is assumed that \( \omega > 0, \alpha_1, \ldots, \alpha_p \geq 0 \) and \( \beta_1, \ldots, \beta_q \geq 0. \)

The parameter \( \alpha_j \) indicates the contributions to conditional variance of the most modern news, the \( \beta_j \) parameter coincides with the moving average segment in the conditional variance, in other words, the recent level of the volatility.

Like in ARCH, \( \sigma_t^2 \) is conditional of the process

\[ \varepsilon_t | \Omega_t \sim N(0, \sigma_t^2). \]

Assume that the process is stationary and compute the unconditional variance of GARCH by taking expectation from both sides of equation for 2.5.

\[ E(\sigma_t^2) = \omega + \sum_{j=1}^{p} \alpha_j E(\sigma_{t-j}^2) + \sum_{j=1}^{q} \beta_j E(\varepsilon_{t-j}^2), \]

and

\[ \sigma_t^2 = \frac{\omega}{1 - \sum_{j=1}^{p} \alpha_j - \sum_{j=1}^{q} \beta_j}. \]  

(2.6)

It means that from the point of view of unconditional variance GARCH process is homoscedastic.
Theorem 1 The GARCH\((p, q)\) process as defined in 2.4 and 2.5 is wide-sense stationary with \(\mathbb{E}(\varepsilon_t) = 0\), \(\text{Var}(\varepsilon_t)\) as in 2.6 and \(\text{Cov}(\varepsilon_t, \varepsilon_s) = 0\) for \(t \neq s\) and only if \(\sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{q} \beta_j < 1\).

Proof: See Bollerslev (1986).

The GARCH\((p, q)\) model may be represented in a form, as an ARMA model on squared perturbations. For this case, let us denote \(\eta_t = \varepsilon_t^2 - \sigma_t^2\), where \(\eta_t \sim \text{i.i.d.} \mathcal{N}(0, 1)\). Replacing \(\sigma_t^2\) by \(\varepsilon_t^2 - \eta_t\) in 2.5, we get

\[
\varepsilon_t^2 = \omega + \sum_{j=1}^{m} (\alpha_j + \beta_j) \varepsilon_{t-j}^2 + \eta_t - \sum_{j=1}^{p} \beta_j \eta_{t-j}, \tag{2.7}
\]

with \(\alpha_j = 0\) for \(j > q\), \(\beta_j = 0\) for \(j > p\).

So, we obtain that 2.7 is an ARMA\((m, p)\) representation for \(\sigma_t^2\) process, where \(m = \max(p, q)\). But 2.7 has an error term, more precisely this is white noise that does not necessarily have constant variance.

2.2.1 Maximum Likelihood Estimation

Maximum Likelihood is a standard method to estimate GARCH\((p, q)\) model. It was introduced by Fisher (1912).

Let observe some \(X_t\). Then

\[
p(X_t) = \frac{1}{\sqrt{2\pi \mathbb{E}(X_t - \mathbb{E}(X_t))^2}} \exp\left(\frac{- (X_t - \mathbb{E}(X_t))^2}{2\mathbb{E}(X_t - \mathbb{E}(X_t))^2}\right) \tag{2.8}
\]

The joint probability of the set of \(T\) observations is

\[
P(X_t) = \prod_{t=1}^{T} \left[ \frac{1}{\sqrt{2\pi \mathbb{E}(X_t - \mathbb{E}(X_t))^2}} \exp\left(\frac{- (X_t - \mathbb{E}(X_t))^2}{2\mathbb{E}(X_t - \mathbb{E}(X_t))^2}\right) \right] \tag{2.9}
\]

2.9 is maximizing the likelihood of observations.

If we take \(\vartheta_-\) - parameter set that maximize the likelihood at each step of time \(t\), based on \(\Omega_{t-1}\), then

\[
\max L(X_t|\vartheta_-, \Omega_{t-1}) = \max(P(X_t)), \tag{2.10}
\]

where \(P(X_t)\) is equal 2.9.

After transformation of 2.10 we get
\[ L_{\log}(X_t|\vartheta_-, \Omega_{t-1}) = - \sum_{t=1}^{T} \log(\E(X_t - E(X_t))^2) - \sum_{t=1}^{T} \left( \frac{(X_t - E(X_t))^2}{\E(X_t - E(X_t))^2} \right) \]  

(2.11)

when ignoring the constant terms.

Considering 2.5 and 2.11, we verify maximum likelihood for GARCH\((p, q)\) model as

\[ L_{\log}(\varepsilon_t|\Omega_{t-1}, \vartheta_-) = - \sum_{t=1}^{T} \log(\sigma_t^2) - \sum_{t=1}^{T} \frac{\varepsilon_t^2}{\sigma_t^2} \]  

(2.12)


### 2.3 GARCH\((1, 1)\) Model

GARCH\((1, 1)\) model is widely-spread and it is very popular in econometrics and financial time series (Hansen and Lunde (2001)).

According to Bollerslev (1986), the GARCH\((1, 1)\) process is given by

\[ y_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \]  

(2.13)

where \( \omega > 0, \alpha_1, \beta_1 > 0. \)

The 2\(m\)th moment for GARCH\((1, 1)\) exists (see Theorem 2 in Bollerslev (1986) paper). So, we write second moment as

\[ \E y_t^2 = \omega + (\alpha_1 + \beta_1) \E y_{t-1}^2 \]  

and stationary condition exist if \( \alpha_1 + \beta_1 < 1 \) and the equation can be rewritten as

\[ \E y_t^2 \equiv \frac{\omega}{1 - \alpha_1 - \beta_1}. \]  

(2.14)

We can also write the 4th moment

\[ \E y_t^4 \equiv \frac{3\omega^2(1 + \alpha_1 + \beta_1)}{(1 - \alpha_1 - \beta_1)(1 - \beta_1^2 - 2\alpha_1\beta_1 - 3\alpha_1^2)} \]  

(2.15)

Now, using 2.14 and 2.15, we can write excess kurtosis coefficient

\[ K \equiv \frac{\E y_t^4}{(\E y_t^2)^2} - 3 = \frac{6\alpha_1^2}{1 - \beta_1^2 - 2\alpha_1\beta_1 - 3\alpha_1^2}. \]  

(2.16)
We determine the autocorrelation function $\rho(k)$ for $GARCH(1, 1)$ model in following way

$$\rho(1) = \frac{\alpha_1 (1 - \alpha_1 \beta_1 - \beta_1^2)}{1 - 2 \alpha_1 \beta_1 - \beta_1^2},$$

(2.17)

$$\rho(k) = (\alpha_1 + \beta_1)^{k-1} \rho(1), \quad k > 1.$$  

(2.18)

### 2.3.1 Forecasting methodology

Historical volatility records of different assets suggest that volatilities are unfeasible to follow random walks, therefore the flat function of forecast coupled with exponential smoothing is unrealistic and unsuitable for the purposes to forecast volatility. We base forecast of the volatility using the $GARCH(1, 1)$ model.

We build volatility forecast referring to Andersen, Bollerslev, Christoffersen and Diebold (2005), Hol and Koopman (2002).

Let us rewrite $GARCH(1, 1)$ given in 2.13 as

$$\sigma_t^2 = (1 - \alpha_1 - \beta_1) \sigma^2 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

(2.19)

where $\sigma \equiv \omega / (1 - \alpha_1 - \beta_1)$ denotes unconditional (or long-run) daily variance. The representation 2.19 displays that the $GARCH$ forecast is constructed as a three elements average.

Going further, we revise 2.13 as

$$\sigma_t^2 = \sigma^2 + \alpha_1 (y_{t-1}^2 - \sigma^2) + \beta_1 (\sigma_{t-1}^2 - \sigma_t^2).$$

(2.20)

Equation 2.20 definitely shows that the $GARCH(1, 1)$ model forecasts (construct adjustments to the variance in current time and to the squared return influence around unconditional variance).

In $GARCH(1, 1)$ model, it is opportune to define a measure about the influence of current news in volatility in the future. To build the volatility term structure (based on the mean-reverting property of $GARCH$ forecast), we compute the expected volatility $k$-day ahead as

$$\sigma_{t+k|t}^2 = \sigma^2 + (\alpha_1 + \beta_1)^{k-1} (\sigma_{t+1}^2 - \sigma^2).$$

(2.21)

Dynamics concerned with volatility forecast using $GARCH(1, 1)$ model gives rich interpretations. In the cases of $GARCH(2, 2)$, for instance, this dynamics may be enriched. In paper of Engle and Lee (1999) is shown that $GARCH(2, 2)$ is out of separate interest.
Chapter 2. Theory and Methods

Correlation

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Negative</th>
<th>Positive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>-0.29 to -0.10</td>
<td>0.10 to 0.29</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.49 to -0.30</td>
<td>0.30 to 0.49</td>
</tr>
<tr>
<td>Large</td>
<td>-1.00 to -0.50</td>
<td>0.50 to 1.00</td>
</tr>
</tbody>
</table>

Table 2.1: Interpretation of correlation size

2.4 Correlation and autocorrelation function

Correlation between two random variables $X$ and $Y$ means the strength and direction of their linear relationship. In probability and statistics it is written as 2.22

$$\rho_{x,y} = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sqrt{E[(X - \mu_X)^2]E[(Y - \mu_Y)^2]}}. \quad (2.22)$$

The correlation between $X$ and $Y$ is defined only if $\text{StDev}X, \text{StDev}Y < \infty$ and $\text{StDev}X, \text{StDev}Y \neq 0$. We should mark that $-1 \leq \rho_{x,y} \leq 1$ and $\rho_{x,y} = \rho_{y,x}$. If $\rho_{x,y} = 0$ means that two random variables are uncorrelated.

Various authors have proposed guidelines for the explanation of a correlation coefficient. Cohen (1988), for instance, has proposed posterior interpretations for correlations, given in the 2.1.

Cohen has mentioned that criteria in 3.1 are not strict. Correlation coefficient depends on the environment and purposes. For instance, 0.9 can show very low correlation if one is examining a physical law using high-quality instruments, but from the other side, may be considered as high in the fields of social sciences.

The correlation matrix of random variables $X_1, \ldots, X_n$ is the $n \times n$ matrix, where $i, j$ element of this matrix is $\rho(X_i, X_j)$. So, we write

$$C = \begin{pmatrix} \rho_{X_1X_1} & \rho_{X_1X_2} & \cdots & \rho_{X_1X_n} \\ \rho_{X_2X_1} & \rho_{X_2X_2} & \cdots & \rho_{X_2X_n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{X_nX_1} & \rho_{X_nX_2} & \cdots & \rho_{X_nX_n} \end{pmatrix}$$

Correlation matrix should be non-negative and definite. It is also symmetric.
2.4.1 Autocorrelation function (ACF)

Consider \( r_t \) as in 2.1. If the linear dependence between \( r_t \) and \( r_{t-l} \) is of interest, then it is useful to explore autocorrelation. The lag-\( l \) autocorrelation of \( r_t \) is the function, which is depending only from \( l \). Especially, we define the lag-\( l \) autocorrelation as

\[
\rho_l = \frac{\text{Cov}(r_t, r_{t-l})}{\sqrt{\text{Var}(r_t)\text{Var}(r_{t-l})}} = \frac{\text{Cov}(r_t, r_{t-l})}{\text{Var}(r_t)},
\]

(2.23)

here we use the property that \( \text{Var}(r_t) = \text{Var}(r_{t-l}) \). It is obvious, that \( \rho_0 = 1 \), \( \rho_l = \rho_{-l} \). Box and Pierce intended the so-called Portmanteau statistic as

\[
Q^*(m) = T \sum_{l=1}^{m} \hat{\rho}_l^2,
\]

(2.24)

where \( T \) - the number of observations, \( \hat{\rho}_l \) - consistent estimate of \( \rho_l \). We define null hypothesis \( H_0 : \rho_1 = \ldots = \rho_m = 0 \) and alternative hypothesis \( H_a : \rho_i \neq 0, i \in 1, \ldots, m \). Then 2.24 is a test statistics for \( H_0 \) against \( H_a \). Ljung and Box altered 2.24 in following way

\[
Q(m) = T(T + 2) \sum_{l=1}^{m} \frac{\hat{\rho}_l^2}{T-l}.
\]

(2.25)

The sample ACF (function of \( \hat{\rho}_1, \hat{\rho}_2 \ldots \)) is suitable for catching the linear dynamics of different types of data.
Chapter 3

Practical Results

3.1 Index Return Data

The data for our empirical studies consists of three Sector Indexes, which are traded on OMX Stockholm Exchange Market.

- OMX Stockholm Energy Price Index (SX1010PI),
- OMX Stockholm Bank Price Index (SX4010PI),
- OMX Stockholm Forest & Paper Price Index (SX151050PI).

We describe the structure of the indices in 4. We use closing prices of each index during the period 5 January 2001 to 13 April 2007. During this period of time we get 1570 values of closing prices. On the Figures 3.1(a), 3.1(b), 3.1(c) we plot investigated daily data (closing prices) for our indexes.

According to these figures we can say that closing prices for all three indexes have tendency to increase. For energy sector the prices change more dynamically with moderate fluctuations over all period, big jumps can be recognized during 2005-2007 years. Through all period closing prices are increasing slowly. As for bank sector, from 2001 up to the beginning of 2004 we can see decreasing of values of closing prices with the small fluctuations. During 2004-2007 we have increasing tendency. The paper sector index is more variable during the hole period. The fluctuations of closing prices are more significant than in bank or energy sectors. Big changes can be observed in 2002, when closing prices changed in interval from 200 to 270. Closing prices are not stable, they always rise or fall evidently.

We have to mention that fluctuations of prices of Energy sector index lie between the values 81.66 to 1533.59; of Bank closing prices are in interval
Chapter 3. Practical Results

Figure 3.1: Closing prices of indexes

(a) Daily Data for Energy PI (2001/01-2007/04)

(b) Daily Data for Bank PI (2001/01-2007/04)

(c) Daily Data for Paper & Forest PI (2001/01-2007/04)
182.18 to 560.83; fluctuations of Paper & Forest closing prices are 166.05 to 285.96 (we mean the minimal and the maximal values of indices).

We calculate daily return series $r_t$ for indexes defined as the first difference between closing prices on consecutive trading days, so

$$r_t = \log p_t - \log p_{t-1}$$

On Figures 3.2, 3.3, 3.4, 3.5, 3.6, 3.7 we graph the daily return series $r_t$ together with the squared return series $r_t^2$ over the full sample period. To each figure we add the summary statistics of both series in tables 3.1, 3.2, 3.3 respectively.

From the figures 3.2 and 3.3 we can say that the most volatile periods were in the beginning of 2001 to the beginning of 2002; in the end of 2006 to beginning 2007 year. From the middle of 2006 up to the end of the observed period the volatility was quite stable and we did not observe some sharp deviations. Due to the analyze of 3.1 we stress that $r_t$ and $r_t^2$ have positive values of skewness. It means that distribution of these series is right-skewed. We observe here positive kurtosis, which shows us that the $r_t$ and $r_t^2$ processes are leptokurtic. We should mention that $r_t^2$ has the value of kurtosis 88.7087 - this fact displays the existence of fat tails and sharp peaks.

Ljung-Box Q(12) statistics indicates that returns are serially uncorrelated, though squared returns display a high degree of serial correlation. Value of $\rho(1)$ which is -0.042 and 0.197 for return series and squared return series respectively indicates the so-called volatility clustering.

Now we will represent statistical results for Bank PI. We also have 1570 observations and we make the statistical analysis for this sector index.

From table 3.2 we observe negative value of skewness for $r_t$. This measure shows us that distribution of return series are right-skewed. As for return distribution we observe left-skewed distribution. We should mention that values, which indicate Q(12) statistics are higher that the values of the same statistics for energy sector. What concern the values of correlation we should say that for daily return series we have only negative correlation coefficients. The level of kurtosis for both series is lower than for Energy Sector Index.

Now we can make valuation for Paper & Forest Sector Index. For this PI we also plot the graphs of daily return series and for squared return series. After that we represent the table with statistical analysis.

According to outcomes from the 3.1, 3.2 and 3.3 for all three indexes we observe that $r_t$ exhibit excess kurtosis. All of the autocorrelation coefficients are differed from zero not more than at 1 % significant level.

\[^1\text{See more in chapter 2}\]
Chapter 3. Practical Results

<table>
<thead>
<tr>
<th>Period $T$</th>
<th>2001–7 (1570 obs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series</td>
<td>$r_t$</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00157</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0071</td>
</tr>
<tr>
<td>St.Deviation</td>
<td>0.02659</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.39213</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.26560</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.17413</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.19270</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>-0.042</td>
</tr>
<tr>
<td>$\rho(2)$</td>
<td>0.002</td>
</tr>
<tr>
<td>$\rho(3)$</td>
<td>-0.011</td>
</tr>
<tr>
<td>$\rho(4)$</td>
<td>-0.069</td>
</tr>
<tr>
<td>$\rho(5)$</td>
<td>-0.013</td>
</tr>
<tr>
<td>$Q(12)$</td>
<td>28.2801</td>
</tr>
</tbody>
</table>

Table 3.1: Summary statistics of Energy PI series

Figure 3.2: The daily return series of Energy PI 2001/01-2007/04

Figure 3.3: The Squared return series of Energy PI 2001/01-2007/04
Correlation between Sector Indexes of OMX Stockholm Exchange Market

<table>
<thead>
<tr>
<th>Period</th>
<th>Series</th>
<th>Mean</th>
<th>St.Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Minimum</th>
<th>Maximum</th>
<th>( \rho(1) )</th>
<th>( \rho(2) )</th>
<th>( \rho(3) )</th>
<th>( \rho(4) )</th>
<th>( \rho(5) )</th>
<th>Q(12)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( r_t )</td>
<td>( r_t^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.053</td>
<td>-0.041</td>
<td>-0.040</td>
<td>-0.008</td>
<td>-0.073</td>
<td>42.6085</td>
</tr>
<tr>
<td></td>
<td>2001-2007 (1570 obs)</td>
<td>0.00030</td>
<td>0.00026</td>
<td>0.01616</td>
<td>-0.04086</td>
<td>4.11675</td>
<td>55.3359</td>
<td>0.01616</td>
<td>0.00000</td>
<td>4.17 \times 10^{-7}</td>
<td>6.29719</td>
<td>-0.04086</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Table 3.2: Summary statistics of Bank PI series

Figure 3.4: The daily return series of Bank PI 2001/01-2007/04

Figure 3.5: The Squared return series of Bank PI 2001/01-2007/04
Table 3.3: Summary statistics of Paper & Forest PI series

<table>
<thead>
<tr>
<th>Period T</th>
<th>2001-2007 (1570 obs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series</td>
<td>$r_t$</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00028</td>
</tr>
<tr>
<td>Variance</td>
<td>0.00015</td>
</tr>
<tr>
<td>St.Deviation</td>
<td>0.01225</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.12308</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.97759</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.05343</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.07312</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>0.061</td>
</tr>
<tr>
<td>$\rho(2)$</td>
<td>0.020</td>
</tr>
<tr>
<td>$\rho(3)$</td>
<td>-0.005</td>
</tr>
<tr>
<td>$\rho(4)$</td>
<td>-0.028</td>
</tr>
<tr>
<td>$\rho(5)$</td>
<td>-0.010</td>
</tr>
<tr>
<td>Q(12)</td>
<td>18.0238</td>
</tr>
</tbody>
</table>

Figure 3.6: The daily return series of Paper & Forest PI 2001/01-2007/04

Figure 3.7: The Squared return series of Paper & Forest PI 2001/01-2007/04
3.2 Volatility Forecast

In this section we observe the daily-time varying $GARCH$ volatility model where we use the term that volatility is explicitly represented as the second moment of daily returns.

For daily-time varying volatility model we consider $GARCH(1,1)$ model, which has been mentioned in chapter 2. We represent $GARCH(1,1)$ model as usual by following equations

$$y = \sigma_t \epsilon_t,$$
$$\sigma_t^2 = \omega + \alpha + \sigma(\sigma_{t-1} \epsilon_{t-1})^2 + \beta \sigma_{t-1}^2,$$

where $\epsilon_t \sim NID(0,1)$, $t = 1, \ldots, T$, $y_t$ denotes the return series of interest which is daily series $R_t$, parameter restrictions are $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ and $\alpha + \beta \geq 1$.

The $GARCH$ parameters, and hence the $GARCH$ volatility, are estimated using exact statistical methods that assist probabilistic inference. Typically we estimate the vector of $GARCH$ parameters $\theta = (\omega, \alpha, \beta)$ by maximizing the log likelihood function

$$\log L(\theta; y_1, \ldots, y_T) \propto -\sum_{t=1}^{T} \left[ \log \sigma_t^2(\theta) - \frac{y_t^2}{\sigma_t^2} \right],$$

which have been discussed before. For $GARCH$ estimation we can refer to Bollerslev (1986), Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994).

$GARCH(1,1)$ process has dynamics that ultimately produce reversion in volatility to a constant long-run value, which enables interesting and practical forecast.

Our practical data as we have noted before consist of 1570 observations over the period from 5 January 2001 to 13 April 2007. We are going to use the first 1548 observations to estimate the parameters of the $GARCH$ model. We left the last 22 meanings of all our observed indices values for forecasting volatility. At first step we apply the $GARCH(1,1)$ model and calculate the parameters and estimations for this model.

Tables 3.4, 3.5, 3.6 give the maximum likelihood estimates for the parameters of $GARCH(1,1)$ model for observed returns series. Represented tables describe statistical results for residuals of the $GARCH(1,1)$ for Energy, Bank and Paper & Forest Indices respectively.
Table 3.4: Outcome for $GARCH$ estimating of Energy Index

According to this table we can infer that estimated coefficients $\omega, \alpha, \beta$ satisfy the requiring conditions, which are $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$. The probability of students $t$-distribution (the last column of the block "Coefficients") allow us to say that confidence interval is really closed to zero value. Three stars indicate goodness of estimated parameters.

In the second block of the table, which is called "Residuals", statistical analysis of residuals for return series $r_t$ for Energy Index is represented. The last block of the Table 3.4 shows Jarque Bera and Box-Ljung tests. These indicators represent good-fit results for estimated parameters and residuals.

Going further, we plot the results for residuals for Energy Index on the Figure 3.8.

From the Figure 3.8 we come to a conclusion that residuals are moved to the right relatively to the Normal distribution. The $Q-Q$ Plot of residuals in the ideal case should be a bisector for I and III quarter of the plane or in other words it should be proper quantiles from a standard normal distribution. In our case $Q-Q$ plot represents quite good results for residuals. So, we conclude from the Figure 3.8 that residuals for Energy Index is satisfied for making further investigations.

Going further, we represent statistical results of estimations and residuals for Bank Sector Index. As we see from the Table 3.5, the meaning of median is not so precise as for the Energy case (for Energy sector it was 0.0000). In analyzing the rest of the Table 3.5 values, we can conclude that the estimated parameters of the $GARCH(1, 1)$ model is fitted for subsequent investigations. On the next step of our research we plot the results of statistical programme
From the Figure 3.9 we can say that for Q-Q plot of residuals we are satisfied by the obtained results. Histogram for residuals shows us that our residuals are closed to the normal distribution.

Then, we proceed to the Paper & Forest Index. As in previous cases we offer the Table 3.6 with statistical analysis.

At the second step we want to forecast volatility using the last 22 values of observed 1570 meaning for each index. It is possible to fix by using the $GARCH(1,1)$ model.

We forecast one-day ahead volatility at time $T$ using the following equation for daily $GARCH(1,1)$ model

$$
\sigma_{t+1}^2 = \hat{\omega} + \hat{\alpha} \gamma_t^2 + \hat{\beta} \sigma_t^2,
$$

where $\hat{\omega}, \hat{\alpha}, \hat{\beta}$ - estimated parameters of the $GARCH(1,1)$ model.

To forecast one-day ahead volatility for the $N$ period of time we apply the law of iterated expectations. It means our forecast is expressed as
### Coefficients

|       | Estimate    | Std.Error | t value | Pr(>|t|)    |
|-------|-------------|-----------|---------|-------------|
| $\omega$ | $4.413 \times 10^{-6}$ | $9.205 \times 10^{-7}$ | 4.794 | $1.63 \times 10^{-6}$ *** |
| $\alpha$ | 0.09599 | 0.01084 | 8.856 | $< 2 \times 10^{-16}$ *** |
| $\beta$ | 0.8871 | 0.0114 | 77.791 | $< 2 \times 10^{-16}$ *** |

### Residuals

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-5.84606</td>
<td>-0.58730</td>
<td>0.04188</td>
<td>0.62815</td>
<td>3.80434</td>
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</table>

### Diagnostic Tests

<table>
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<tr>
<th>Test</th>
<th>Data</th>
<th>$X^2$</th>
<th>df</th>
<th>$p$ - value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque Bera Test</td>
<td>Residuals</td>
<td>208.3351</td>
<td>2</td>
<td>$&lt; 2.2 \times 10^{-16}$</td>
</tr>
<tr>
<td>Box-Ljung test</td>
<td>Sq. Residuals</td>
<td>0.2581</td>
<td>1</td>
<td>0.6114</td>
</tr>
</tbody>
</table>

Table 3.5: Outcome for GARCH estimating of Bank Index

\[
\sigma^2_{t+1,t+N} = \sum_{j=1}^{N} \frac{\hat{\omega}}{1-\hat{\alpha}-\hat{\beta}} + (\hat{\alpha} + \hat{\beta})^{-1} \left( \sigma^2_{T+1} - \frac{\hat{\omega}}{1-\hat{\alpha}-\hat{\beta}} \right).
\]

The expression above can be rewritten in the simpler form as

\[
\sigma^2_{t+N} = \frac{\hat{\omega}}{1-\hat{\beta}} = \sum_{j=1}^{N} \hat{\beta}^{-1} y^2_{T-j}.
\]

According to the last equation for forecasting volatility one-day ahead, we compute volatility for all indices with $N = 22$. To reach this aim we use R Software (see programme code in 4). We put obtained results in Table 3.7 together with the statistical analysis for forecasting volatility. According to this table we see that the lowest volatility value is observed for Paper & Forest Index, the highest value is observed for Energy Index. As regards the variance, we can mention that value for three indices are differed from each other not more than 0.3. All volatility series are positively skewed, have negative values of kurtosis.

As further step, we plot volatility series to gain an insight how they are look like and how they behave during our forecasting period. To next figures we also add graphs of closing prices over the period from 13 March 2007 to 13 April 2007.

### 3.3 Correlation

#### 3.3.1 Correlation with respect to volatility

Let us denote $X_1, X_2, X_3$ - vectors consist of forecasting volatility values for Energy, Bank and Paper & Forest Indices respectively over the time period...
Figure 3.9: Statistical summary of GARCH(1,1) estimations. Bank Index from 13 March 2007 to 13 April 2007. The dimension of each vector is equal 22. $X_1$ denotes volatility for Energy Index, $X_2$ - for Bank Index, $X_3$ - for Paper & Forest Index.

According to the theory (see Chapter 2) we define correlation matrix of indices in the following way

$$C = \begin{pmatrix}
\rho(X_1 X_1) & \rho(X_1 X_2) & \rho(X_1 X_3) \\
\rho(X_2 X_1) & \rho(X_2 X_2) & \rho(X_2 X_3) \\
\rho(X_3 X_1) & \rho(X_3 X_2) & \rho(X_3 X_3)
\end{pmatrix}$$

where $\rho(X_i, X_j) = \frac{\text{Cov}(X_i, X_j)}{\text{StDev}_{X_i} \text{StDev}_{X_j}}$, $i, j = 1, 2, 3$.

Using R Software (see Appendix) we calculate standard deviation, covariance function of $X_1, X_2, X_3$. Going further, we compute correlation between $X_1, X_2, X_3$. As a result we obtain the following outcomes and put them into the 3.8.

From the Table 3.8 we can say that the lowest level of volatility correlation is observed between Energy and Paper & Forest Indices and is equal to 0.85 approximately. The highest volatility correlation value is observed between Bank and Paper & Forest Indices.
Chapter 3. Practical Results

| Coefficients | Estimate | Std.Error | t value | Pr(>|t|) |
|---------------|----------|-----------|---------|----------|
| $\omega$      | $8.603 \times 10^{-6}$ | $1.434 \times 10^{-6}$ | 5.998 | $2.00 \times 10^{-9}$ |
| $\alpha$      | 0.1252   | 0.01694   | 7.387   | $1.50 \times 10^{-13}$ |
| $\beta$       | 0.8198   | 0.02216   | 37.000  | $< 2 \times 10^{-16}$ |

<table>
<thead>
<tr>
<th>Residuals</th>
<th>Min</th>
<th>1Q</th>
<th>Median</th>
<th>3Q</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-4.671</td>
<td>-0.5490</td>
<td>0.0000</td>
<td>0.598</td>
<td>6.382</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Diagnostic Tests</th>
<th>Data</th>
<th>X^2</th>
<th>df</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jarque Bera Test</td>
<td>Residuals</td>
<td>321.2912</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Box-Ljung test</td>
<td>Sq. Residuals</td>
<td>0.002</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.6: Outcome for GARCH estimating of Bank Index

<table>
<thead>
<tr>
<th>Volatility series, $\sigma_t$</th>
<th>Mean</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Index</td>
<td>29.402</td>
<td>22.510</td>
<td>0.078</td>
<td>-1.652</td>
<td>22.386</td>
<td>36.370</td>
</tr>
<tr>
<td>Bank Index</td>
<td>20.689</td>
<td>22.344</td>
<td>0.342</td>
<td>-1.712</td>
<td>14.939</td>
<td>28.521</td>
</tr>
<tr>
<td>Paper &amp; Forest Index</td>
<td>18.193</td>
<td>22.857</td>
<td>0.624</td>
<td>-1.183</td>
<td>13.252</td>
<td>27.328</td>
</tr>
</tbody>
</table>

Table 3.7: Out-sample results for volatility forecasting over 13 March 2007 to 13 April 2007

Going further, we construct correlation matrix of forecasting volatility measures for Indices. This matrix is represented by the following

$$C = \begin{pmatrix}
1 & 0.9430883 & 0.8495505 \\
0.9430883 & 1 & 0.9503548 \\
0.8495505 & 0.9503548 & 1
\end{pmatrix}$$

As we can observe, the correlation between Sector Indices, namely Energy, Bank and Paper & Forest, is closed to one and is not closed to zero. It means that volatilities of investigated indices are linearly dependent. Also, it is essential to mention that the value of obtained correlation is positive. This indicates that if volatility of one presented indices will increase, other will follow the same movement or vice versa.
Correlation between Sector Indexes of OMX Stockholm Exchange Market

Figure 3.10: Statistical summary of GARCH(1, 1) estimations. Paper & Forest Index

<table>
<thead>
<tr>
<th></th>
<th>Cov($X_i, X_j$)</th>
<th>$\rho(X_i, X_j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X₁, X₂)</td>
<td>19.27084</td>
<td>0.9430883</td>
</tr>
<tr>
<td>(X₁, X₃)</td>
<td>21.15092</td>
<td>0.8495505</td>
</tr>
<tr>
<td>(X₂, X₃)</td>
<td>21.47773</td>
<td>0.9503548</td>
</tr>
</tbody>
</table>

Table 3.8: Covariance and correlation between Sector Indices

3.3.2 Autocorrelation with respect to volatility

Using R Software we compute autocorrelation function for investigated indices and put them in the next tables.
Chapter 3. Practical Results

(i) Volatility forecast for Energy Index 2007/03/13-2007/04/13


(iii) Volatility forecast for Bank Index 2007/03/13-2007/04/13

(iv) Closing prices for Bank Index 2007/03/13-2007/04/13

(v) Volatility forecast for Paper & Forest Index 2007/03/13-2007/04/13

(vi) Closing prices for Bank Index 2007/03/13-2007/04/13
Figure 3.11: ACF for forecasting volatility (i)-Energy PI, (ii)-Bank PI, (iii)-Paper & Forest PI
Table 3.9: ACF for lag 1–12 of Energy PI, Bank PI and Paper & Forest PI
Chapter 4

Conclusions

In this paper we examine $GARCH(1, 1)$ model as a useful instrument for analyzing and forecasting volatility of price indices. We evaluate our investigations on the basis of daily time-varying volatility $GARCH(1, 1)$ model. We conclude that the $GARCH(1, 1)$ model is satisfactory suitable for residuals and forecasts.

We observe three sector indices of OMX Stockholm Exchange Market: Energy, Bank and Paper & Forest over the period of time from 5 January 2001 to 13 April 2007. For each sector we forecast one-day ahead volatility using the $GARCH(1, 1)$ model and we compute the correlation between investigated indices with respect to forecasting volatility. Our results are surprising, because we have expected that the highest level of correlation will be between Paper&Forest and Energy Indices. But we get the highest level of correlation between Bank and Paper&Forest Indices.

We conclude that all three indices have quite high level of correlation. It means, that from the investors side due to decrease the risk level of their portfolios, they should reallocate their assets taking into consideration the high correlation of Bank, Energy and Paper&Forest Indices.
Notation

\((\Omega, \mathcal{F}, \{\mathcal{F}_n\}, P)\) filtered probability space with set of outcomes \(\Omega\), sigma algebra \(\mathcal{F}\), flow or filtration \(\{\mathcal{F}_n\}\) and probability measure \(P\).

\(P(A)\) Probability of the event \(A\)

\(\mu_X\) Mean of the random variable \(X\).

\(E(X), \text{Var}(X)\) Expectation and variance of the random variable \(X\).

\(\text{Cov}(X,Y)\) Covariance of the random variables \(X\) and \(Y\) with respect to the measure \(\mu\).

\(\{X_t: t \in T\}\) A random process with time index set \(T\). \(\{X_t\}\) is said to be a process in discrete time if \(T\) is discrete and a process in continuous time if \(T\) is continuous.

\(\Phi(x), \phi(x)\) Normal distribution and density values respectively at point \(x\).

\(\mu(t)\) predictable and finite variation process.

\(M\) local martingale.

\(M^c\) infinite variation local martingale component.
Bibliography


Appendix

1. Description of Sector Indices OMX Stockholm Exchange Market


This Sector Index includes following Companies:

- Broström B
- Concordia Maritime B
- Lundin Petroleum
- PA Resources
- Swithoid Tankers B
- Tanganyika Oil Company SDB
- Vostok Nafta SDB

2. OMX Stockholm Banks Price Index (2001-2007)

This Sector Index includes following Companies:

- Nordea Bank
- SEB A
- SEB C
- Sv. Handelsbanken A
- Sv. Handelsbanken B
- Swedbank A


This Sector Index includes following companies:

- Bergs Timber B
- Billerud
- Holmen A
- Holmen B
- Rottneros
2. Program code of R software for statistical analysis of real data.

```r
data<-read.csv("filename", sep=";", header=T)
data
data1<-as.matrix(data)
data1
data2<-data1[,3]
data2
library(graphics)
plot(data2, type='l',ylab="value",xlab="index")
library(stats)
i<-1:1570
returns<-log(data2[i+1]/data2[i])
mean(returns)
var(returns)
sd(returns)
library(vars)
library(e1071)
skewness(returns)
kurtosis(returns)
Box.test(returns)
min(returns)
max(returns)
squaredreturns<-returns ^2
mean(squaredreturns)
var(squaredreturns)
sd(squaredreturns)
library(vars)
library(e1071)
skewness(squaredreturns)
kurtosis(squaredreturns)
min(squaredreturns)
max(squaredreturns)
Box.test(squaredreturns)
```

3. Program code of R Software for estimating parameters of \( GARCH(1,1) \) Model

```r
datagarch1<-returns[1:1548]
datagarch<-garch(datagarch1,order=c(1,1))
```
summary(datagarch)
plot(datagarch,type='l'))

4. Program code of R software for forecasting volatility

volatility<-function(s,c1,c2,t,n) {
library(tseries)
library(fSeries)
library(fBasics)
library(stats)
z<-read.table(s,sep="\t",dec=",",header=F)
x<-ts(diff(log(z[,c2])))
z[,c1]
q<-garch(x, order=c(1,1))
B<-rep(0,times=22)
for(i in 1:22)
  {B[i]=q$coef[3]^(i-1)}
first<-length(z)-22
last<-length(z)-22-21
Vol<-rep(0,times=22)
r<-diff(log(z))
for(j in 1:22){
  R<-r[first:last]^2
  first<-first+1 +last<-last+1
  h<-sqrt(Vol*252)*100
ts(h)
plot(h, type='l')
}

5. Program code of R software for computing volatility correlation

h<-en
h<-bn
h<-pn
a<-sd(en)
b<-sd(bn)
c<-sd(pn)
cov(x1,x2)/(sd(x1)*sd(bn))
cov(en,pn)/(sd(en)*sd(pn))
cov(bn,pn)/(sd(bn)*sd(pn))
mean(en)
var(en)
sd(en)
skewness(en)
kurtosis(en)
min(en) max(en)