Pulse Energy Stabilization of a Femtosecond Laser Pulse Chain

Mallikarjun Madagalam

Department of Physics,
Division of Atomic Physics,
Lund University, January 17, 2017.
Mallikarjun Madagalam

Pulse Energy Stabilization of a Femtosecond Laser Pulse Chain, © May 2016

SUPERVISORS:
Cord Arnold (Lund University)
Miguel Miranda (Lund University)
Pererik Andreaasson (Halmstad University)
Lund, May 2016

1 Cover Image: Femtosecond laser pulse generated in Lund laser center (LLC).
In the end, it’s not the years in your life that count.
   It’s the life in your years
   — Abraham Lincoln

Dedicated to my parents
Pulse energy stabilization of a femtosecond laser pulse chain

A stabilization system was designed and implemented in order to reduce the power fluctuations of a Ti:Sapphire ultrafast laser at Lund Laser Center, Lund University. This laser produces a train of femtosecond laser pulses. An active stabilization system was designed using an acousto-optic modulator (AOM), a large photodiode (PD), an active analog low pass filter and a controller with feedback. AOM deviates some energy of the laser beam by Bragg diffraction principle and the zero order beam was detected by the photodiode. Large area PD was used in order not to damage the detector. This measured signal was shaped and spread over time to make the pulse intensity measurement possible by using an analog active low pass filter and controlled by a simple PI controller with feedback. The results showed that the active attenuation and filter is working as expected with minimum amount of fluctuations. An analog PI controller was designed but the experiment was not performed. Hence, the laser system was not stabilized well enough. Due to the timing and technical constraints additional steps were not taken, some of the other possible solutions have been proposed and the complete system performance might be studied in near future, it might be available for future studies.
Everybody is a genius.
But if you judge a fish by it's ability to climb a tree, it will live it's whole life believing that it is stupid
— Albert Einstein

ACKNOWLEDGEMENTS

I would like to thank my adviser Cord Arnold for his immense knowledge and experience. Who has helped me throughout my work, the door to his office was always open whenever I had a problem or doubt about my thesis work. He consistently allowed me to work on my own ideas and kept me in the right direction whenever I needed.

I would also like to thank my co-adviser Miguel Miranda who was always right beside me. I am always greatful for his patience and advises given to me whenever I asked him to do so. I would always remember him for what he has done to finish my work.

I want to thank my supervisor Pererik Andreasson from Halmstad University who was also my program manager. He has helped me in each and every aspect of my thesis work as well as in this master’s program.

I like to express my gratitude to Anne L´Huillier and to everyone who welcomed me in the attosecond physics group. I am thankful for their aspiring guidance and invaluably constructive criticism and friendly advise during this project work.

Last but not least, I would like to thank my parents and friends whoever involved all along my work.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>BACKGROUND</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>1.1</td>
<td>Research Aim</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>BACKGROUND</td>
<td>7</td>
</tr>
<tr>
<td>2.1</td>
<td>Description of Short pulses</td>
<td>7</td>
</tr>
<tr>
<td>2.1.1</td>
<td>Pulse Characteristics</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>Carrier Envelope Phase Stabilization</td>
<td>9</td>
</tr>
<tr>
<td>2.2.1</td>
<td>CEP in Time domain</td>
<td>9</td>
</tr>
<tr>
<td>2.2.2</td>
<td>CEP in Frequency Domain</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>Dependency of CEP on Pulse Energy</td>
<td>12</td>
</tr>
<tr>
<td>ii</td>
<td>METHODOLOGY</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>PULSE ENERGY STABILIZATION: SYSTEM DESIGN AND IMPLEMENTATION</td>
<td>17</td>
</tr>
<tr>
<td>3.1</td>
<td>Stabilization System Design</td>
<td>17</td>
</tr>
<tr>
<td>3.2</td>
<td>Working principles of important blocks</td>
<td>17</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Acousto-Optic Modulator (AOM)</td>
<td>17</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Photodiode</td>
<td>20</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Analog Low pass filter</td>
<td>20</td>
</tr>
<tr>
<td>3.2.4</td>
<td>PID Controller Design</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>RESULTS AND DISCUSSIONS</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>CONCLUSIONS</td>
<td>31</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>iii</td>
<td>APPENDIX</td>
<td>35</td>
</tr>
<tr>
<td>A</td>
<td>LABORATORY INFORMATION</td>
<td>37</td>
</tr>
<tr>
<td>B</td>
<td>HIGH HARMONIC GENERATION (HHG)</td>
<td>39</td>
</tr>
<tr>
<td>C</td>
<td>CHIRPED PULSE AMPLIFICATION (CPA)</td>
<td>41</td>
</tr>
<tr>
<td>C.1</td>
<td>Chirped Gaussian Pulse</td>
<td>41</td>
</tr>
<tr>
<td>D</td>
<td>ANALOG ELECTRONIC CIRCUITS</td>
<td>43</td>
</tr>
<tr>
<td>ACRONYMS</td>
<td>EXPLANATION</td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------------------------</td>
<td></td>
</tr>
<tr>
<td>LLC</td>
<td>Lund Laser Center</td>
<td></td>
</tr>
<tr>
<td>CEP</td>
<td>Carrier Envelope Phase</td>
<td></td>
</tr>
<tr>
<td>CEO</td>
<td>Carrier Envelope Offset</td>
<td></td>
</tr>
<tr>
<td>AOM</td>
<td>Acousto Optic Modulator</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>Proportional Integral Derivative</td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>Proportional Integral</td>
<td></td>
</tr>
<tr>
<td>SSE</td>
<td>Steady State Error</td>
<td></td>
</tr>
<tr>
<td>CPA</td>
<td>Chirped Pulse Amplification</td>
<td></td>
</tr>
<tr>
<td>HHG</td>
<td>High Harmonic Generation</td>
<td></td>
</tr>
<tr>
<td>XUV</td>
<td>Extreme Ultraviolet Radiation</td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>Photo Diode</td>
<td></td>
</tr>
<tr>
<td>VCO</td>
<td>Voltage Controlled Oscillator</td>
<td></td>
</tr>
<tr>
<td>VVA</td>
<td>Voltage Variable Attenuator</td>
<td></td>
</tr>
<tr>
<td>RPS</td>
<td>Regulated Power Supply</td>
<td></td>
</tr>
<tr>
<td>GDD</td>
<td>Group Delay Dispersion</td>
<td></td>
</tr>
</tbody>
</table>
Part I

BACKGROUND
INTRODUCTION

A stable optical short pulse source is very important for realizing future ultra high speed optical communication. Fiber-Optic communication using lasers is a key technology in modern communications, allowing services such as the Internet. One of the most potential techniques to realize this kind of sources is mode locking of Ti:Sapphire lasers for producing femtosecond laser pulses with kHz repetition rates in the 800 nm wavelength region [cf. Appendix A]. Ti:Sapphire lasers are tunable lasers which emit red and near-infrared light in the range from 650 to 1100 nm. These lasers are mainly used in scientific research because of their tunability and their ability to generate ultrashort pulses. Pulse durations of a few femtoseconds in the visible are approaching the fundamental pulse-width limitation of roughly one optical cycle (roughly one wavelength in spatial extent). The key to attosecond pulse generation is the use of highly nonlinear optical frequency-conversion methods to produce radiation at much higher frequencies (much shorter wavelengths), corresponding to extreme ultraviolet (XUV) and x-ray spectral regions. At such frequencies the duration of a single optical cycle is reduced, making attosecond pulses possible. At these time scales the motion of bound electrons that mediate important laser-matter interactions may usually be viewed as instantaneous. Conversely, attosecond time scales, XUV and x-ray frequencies bring in entirely new physics in which laser-matter interactions are sensitive to the non-instantaneous dynamics of bound electron motion[1].

Since the beginning of High Harmonic Generation (HHG) experiments in 1977[2] the continuous improvement of laser technologies has made it easy to realize new spectrum of experiments. The generation of attosecond or XUV laser beams is possible based on the principle of HHG [cf. Appendix B]. High-harmonic generation by focusing a femtosecond laser onto a gas jet is a well-known method of producing coherent extreme-ultraviolet (XUV) light[3, 4, 5]. This nonlinear conversion process requires high pulse intensities, greater than $10^{13}$ W·cm$^{-2}$, which are not directly attainable using only the output power of a femtosecond oscillator. Chirped-pulse amplification (CPA)[cf. Appendix C] enables the intensity to exceed this threshold by incorporating several regenerative and multi-pass amplifier cavities in tandem[6].

Ultrashort pulses are mainly used in the study of atomic and molecular processes because of the fast nature of electrons, and for imaging microscopic structures such as DNA. Ultrafast optics technology is important not only for pulse generation but also for signal processing, for data detection, and for the advanced metrology necessary for characterizing and optimizing ultrashort-pulse transmission. With femtosecond lasers, materials processing is possible using lower
pulse energies, due to the very high peak powers, which lead to new physical mechanisms[1].

As we discussed above, in order to produce a coherent extreme-ultraviolet (XUV) light by HHG process an ultrashort pulse source with high intensity is required. Most importantly, when we are working on HHG process, the driving laser should be carrier envelop phase (CEP) stabilized. The absolute carrier phase is normally not important in optics; however, for such ultrashort pulses, it can have physical consequences[7].

Among all available high power laser facilities in Lund laser center (LLC)[cf. Appendix A], the kHz laser is mostly used by attosecond physics research group. The whole work has been done under this group in atomic physics division, physics department. This kHz laser has the ability to produce 20 fs laser pulses with 5 mJ of energy at 1 kHz repetition rate, and is composed of seed oscillator with chirped pulse amplification stages. The kHz laser system is centered around the 800 nm wavelength region.

Today, High frequency electronics like microwave and RF technology is more pervasive than ever. This is especially true in the commercial sector, where modern applications include cellular telephones, 3G and WiFi wireless networking, global positioning systems, ultra wideband radio and radar systems, and microwave remote sensing systems for the environment. Defense systems continue to rely heavily on microwave technology for passive and active sensing, communications, and control systems. There should be no shortage of challenging problems in RF and microwave engineering in the foreseeable future, and there will be a clear need for engineers having both an understanding of the fundamentals of microwave engineering and the creativity to apply this knowledge to problems of practical interest[8].

Modern RF and microwave engineering predominantly involves distributed circuit analysis and design, in contrast to the waveguide and field theory orientation of earlier generations. The majority of microwave engineers today design planar components and integrated circuits without direct recourse to electromagnetic analysis[8].

1.1 RESEARCH AIM

The aim of this project is to implement a stabilization system, that can reduce the power fluctuations of a Ti:Sapphire kHz laser system, which is present in attosecond physics lab in Lund laser center. This is an optoelectronics project, which includes attenuation of the femtosecond laser pulses, detection and conversion to an electrical signal (voltage or current) using a transducer which can convert a signal from one form (light signal) to other (electrical signal). This electrical signal will be shaped and controlled in a closed loop feedback system.

One of the reasonable guesses for these power fluctuations (It could be seen as a hum below 100Hz which is just a technical guess) is the cryo-cooled Ti:Sapphire crystal which vibrates due to the pressure in-
side its cooling crystals and it is difficult to measure the precise fluctuations prior to the experiment. A system which can deliver a long-term stability and reliability has to be designed and implemented in order to correct these fluctuations. This laser system is mostly used in intensity dependent high harmonic generation processes to generate attosecond pulses in a gas medium.

The upcoming chapters provide the complete picture of this research project. The background, design and implementation of the stabilization system, results and discussions and finally the future expectations from this project will be presented respectively.
Interest in ultrashort optical pulses began with the invention of the laser and has been one of continuous progress toward shorter and shorter time scales. The earliest solid-state and semiconductor lasers were naturally pulsed, and the development of CW lasers required significant additional effort. The development of nanosecond pulses was followed by picosecond pulses, which ultimately led to femtosecond pulses, and more recently to attosecond pulses.

When applied to optics, the terms ultrafast and ultrashort generally describe pulses of widths in the picosecond to femtosecond, or shorter, regimes. In electronics, however, these terms refer to pulses of nanosecond to tens of picosecond widths since the ultimate speed limit of electronics is well below optics. A nanosecond electrical pulse has a GHz spectral width and must be guided by a broadband microwave circuit. A picosecond electrical pulse has a THz spectral width, which cannot be sustained by conventional electrical or microwave circuits. If a femtosecond electrical pulse were to be generated, it would cover a spectral band of hundreds of THz, which equals the entire frequency range extending from 0 Hz to the edge of the visible band (approx. 0.3 µm). Additionally, by virtue of the uncertainty principle $\Delta E \Delta t \geq \hbar/2$, such a pulse would have an energy uncertainty exceeding 1.5 eV, i.e., roughly the magnitude of the bandgap energy in typical semiconductors, which would make conventional electronics unreliable.

2.1 DESCRIPTION OF SHORT PULSES

Ultrashort optical pulses may be generated by a combination of specially designed lasers employing various switching techniques or mode locking methods. The pulses generated by the lasers will be further compressed and reshaped by use of special techniques based on linear and nonlinear dispersive optical components and systems. A simulated femtosecond pulse as shown in figure 1 with time constant $\tau=15$ fs and Full Width Half Maximum (FWHM) of $\tau_{\text{FWHM}}=17.7$ fs.

2.1.1 Pulse Characteristics

A pulse of light is described by an optical field of finite time duration. We use a scalar theory and represent the field components with a generic complex wavefunction $U(r,t)$ normalized such that the optical intensity $I(r,t)=|U(r,t)|^2 (\text{W/m}^2)$. 
2.1.1.1 Temporal Representation

The complex wavefunction describing an optical pulse of central frequency $\nu_0$ is written in the form

$$U(t) = A(t)e^{j\omega_0 t}$$

(1)

where $A(t)$ is the complex envelope and $\omega_0 = 2\pi \nu_0$ is the central angular frequency. A more general Gaussian pulse has a complex envelope

$$A(t) = A_0e^{-\alpha t^2}$$

(2)

where $\alpha = (1-j\alpha)/\tau^2$ is a complex parameter and $\tau$ and $a$ are real parameters, so that

$$A(t) = A_0e^{-t^2/\tau^2}e^{jat^2/\tau^2}$$

(3)

The complex envelope itself is characterized by its magnitude $|A(t)|$ and phase $\varphi(t) = \text{arg}(A(t))$, so that

$$U(t) = |A(t)|e^{j(\omega_0 t + \varphi(t))}.$$  

(4)

The optical intensity $I(t) = |U(t)|^2 = |A(t)|^2(W/m^2)$ and the area under the intensity function $\int I(t)dt$ is the energy density (J/m^2). The intensity profile of a typical Gaussian function, $I(t) \propto e^{-2t^2/\tau^2}$. Figure 2 illustrates the temporal functions that characterize an optical pulse.

From Chapter 1, we know that carrier phase is not really important in optics but, in the case of ultrashort pulses it can show some physical consequences[7]. Then it becomes really essential to stabilize the phase of the driving laser. Coming sections describe what is this phase and how it could be stabilized in detail.
The carrier-envelope phase (CEP) or carrier-envelope offset (CEO) phase shows a great significance when the laser pulse duration becomes smaller as picoseconds or lesser.

2.2.1 CEP in Time domain

Mathematically, the electric field of a pulse can be expressed as

\[ E(t) = A(t) \cos(\omega_c t + \phi_{CE}) \]  

where \( A(t) \) is the pulse envelope, \( \omega_c \) is the carrier frequency, and \( \phi_{CE} \) is the carrier-envelope phase as shown in figure 3.
The reliable periodicity of a train of optical pulses generated by a mode-locked laser allows identification of a phase referenced to the pulse envelope. Relative to this frame, the phase ($\phi_{CE}$) of the oscillating electric field can vary (cf, figure 4(a)), depending on conditions both within and outside the laser cavity. In general, this phase is not constant from pulse to pulse because the group and phase velocities differ inside the laser cavity. For a clear understanding of the dynamics of $\phi_{CE}$, the CEP can be broken into two components,

$$\phi_{CE} = \phi_0 + \Delta \phi_{CE}$$  \hspace{1cm} (6)

where $\phi_0$ is the “static” offset CEP and $\Delta \phi_{CE}$ represents the pulse-to-pulse change in CEP due to conditions inside the cavity of the laser oscillator. As the pulse propagates through any medium outside the laser cavity (except vacuum), a difference between the phase and group velocities\(^1\) (caused by dispersion) will cause $\phi_0$ to vary; so in reality, $\phi_0$ is not truly static. In a similar way, the physical origin of $\Delta \phi_{CE}$ results from dispersion of the optical elements inside a laser cavity\[^{10}\].

It is now possible to detect and control $\Delta \phi_{CE}$. The techniques used to stabilize the pulse to pulse evolution of CEP is best understood in frequency domain. The next section would describe the CEP in frequency domain and the steps to stabilize the CEP offset.

2.2.2 CEP in Frequency Domain

The spectrum of a mode-locked laser is characterized by two radio frequencies (rf). One of these frequencies is the laser repetition rate, $f_{rep}$, which determines the comb spacing. The second is the offset frequency, $f_0$, which determines the absolute position of the comb as shown in figure 4(b). As a result the frequency of the $n^{th}$ comb line, $\nu_n$, is represented by:

$$\nu_n = nf_{rep} + f_0$$  \hspace{1cm} (7)

here, $n$ is a large integer multiplying $f_{rep}$ up into the optical regime. The entire comb is offset from exact harmonics of $f_{rep}$ by an offset frequency ($f_0$). From a careful derivation\[^{11}\], the relation between $f_0$ and $\Delta \phi_{CE}$ can be expressed as

$$\Delta \phi_{CE} = 2\pi \frac{f_0}{f_{rep}}$$  \hspace{1cm} (8)

Thus, the task of stabilizing $\Delta \phi_{CE}$ has been reduced to the stabilization of $f_0$. Therefore, by controlling $f_0$ and $f_{rep}$, $\Delta \phi_{CE}$ can be controlled.

---

\(^{1}\) Group velocity is velocity of the envelope and phase velocity is the velocity at which the phase of a wave propagates.
2.2 CARRIER ENVELOPE PHASE STABILIZATION

Figure 4: Time-frequency correspondence and relationship between $\Delta \phi_{CE}$ and $f_0$. (a) In the time domain, the relative phase between the carrier and the envelope evolves from pulse to pulse by an amount of $\Delta \phi_{CE}$. (b) In the frequency domain, the elements of the comb of a mode-locked pulse train spaced by $f_r$.

As each comb element is shifted by the (same) offset frequency by optically heterodyning different harmonics of the frequency comb together, it is not possible to extract the value of the offset frequency. Instead, scaling of the comb spectrum must be implemented before the heterodyne comparison. A straightforward method is to frequency double the red end of the comb spectrum and compare it with the existing spectrum at the blue end where these two beams spectrally overlap. Thus, the simplest heterodyne procedure requires an octave of optical bandwidth and is generally known as $\nu$-to-$2\nu$ self referencing technique\[10\]. This is the standard way to detect CEP drift. Some experiments performed in Lund Laser Center\[12\] and many other places in the world have proved that HHG process is highly dependent on electric field intensity and phase. From figure 5, it proves that the influence of phase is significant. Figure 5b presents two lineouts from figure 5a with a change of CEP of $\Delta \phi = \pi/2$ between them. Three different regions can be distinguished: in region I, in the cut-off (above 40eV), spectral peaks shift from odd to even harmonics as the CEP changes by $\pi/2$. In region III (low order harmonics), the position of the harmonic peaks does not change, but their amplitude depends on the CEP. In between, in region II, the harmonic spectrum shows non trivial dynamics upon CEP change; spectral peaks are not located at multiple orders of the laser frequency and their position shift with CEP\[12\].
A CEP stabilization system was implemented and installed in kHz laser system in 2009\cite{13}, based on this ν-to-2ν self referencing technique. Which is very simple to implement, robust and offers a visually clear picture of the phase stability (the jitter of the spectral fringe pattern)\cite{14}. A clear reduction in CEP jitter has been seen thus, in order to increase the stability further some other additional steps has to be taken. To further reduce the CEP jitter, additional layers of vibration damping material might be put under the pump lasers and also under the pump legs, active stabilization of pulse energy fluctuations are other minimally invasive steps that can be taken for further reduction of noise\cite{13}.

2.3 DEPENDENCY OF CEP ON PULSE ENERGY

When the duration of high-power laser pulses approaches a single optical cycle, the CE phase of the pulses starts to play a major role in laser-matter interactions. For pulses from laser amplifiers, the CE phase change from one laser shot to the next can be measured optically by using ν-to-2ν interferometers. The measured phase variation between successive pulses can be used as a feedback control signal to stabilize the CE phase of the amplified pulses, provided that the oscillator phase is also locked. It has been expected that the ampli-
fier CE phase measurement and stabilization are influenced by laser pulse energy fluctuations[15]. The coupling coefficient between the CE phase and the laser energy, is determined by:

\[ C_{PE} = \Delta \phi_{CE} / (\Delta \epsilon / \epsilon) \]  

(9)

Where \( \Delta \phi_{CE} \) is the CE phase change of the output pulse caused by a relative laser energy change \( \Delta \epsilon / \epsilon \) in the in-loop \( \nu \)-to-2\( \nu \) interferometer.

An experiment was conducted in J.R.MacDonald Laboratory, using the Kansas Light Source laser system, which is equipped with grating-based stretchers and compressors[15]. The CE phase to laser energy coupling coefficient was determined by modulating the pulse energy in the in-loop \( \nu \)-to-2\( \nu \) interferometer while measuring the CE phase variation with an out-loop interferometer. The results are shown in figure 6. This results have shown the dependency of CEP on the pulse energy variations. The measured coupling coefficient between the CE phase and pulse energy change was 160 mrad per 1% of energy fluctuations. This result is very important to measure and stabilize the CE phase of high power laser pulses. This gives the motivation to perform this research on pulse energy fluctuations in order to reduce the influence of pulse power on CEP stabilization[15].
Part II

METHODOLOGY
3.1 STABILIZATION SYSTEM DESIGN

An active feedback loop stabilization system has been designed based on a simple principle as shown in figure 7. Working of every block will be explained in upcoming sections in detail. The principle of the stabilization system is as follows. After all the chirped pulse amplification (CPA) stages, an acousto-optic modulator (AOM) was placed. Here, AOM diverts some of the energy from the beam path by diffraction principle. The zeroth order beam was passed through an aperture and would be detected by a large area photodiode (PD). Large area PD was used in order not to damage the PD because of the high intensity of the laser beam. Photodiode would convert the laser light into an electrical signal (either voltage or current). This signal was shaped by an active analog low pass filter, the filtered output signal was sent as an input to a PI controller and the output voltage signal was given as feedback to the AOM driver.

This system has been placed in order not to affect any other experiments, which are going on simultaneously.

3.2 WORKING PRINCIPLES OF IMPORTANT BLOCKS

3.2.1 Acousto-Optic Modulator (AOM)

Sound can change the refractive index of an optical medium. This means, the modification of the refractive index of a medium by mechanical vibrations of a sound wave. This effect of sound on an optical medium is known as acousto-optic effect. Many useful devices make

Figure 7: Schematic diagram of the stabilization system with different results indicated by numbers.
use of this effect, which include optical modulators, switches, deflec-
tors, filters, isolators, frequency shifters, and spectrum analyzers[9].

Acousto-optic modulator is a device which can modulate the fre-
quency, intensity and direction of a laser beam. Acousto-optic modu-
lators are sometimes called as Bragg Cells because they work based
on the principle of Bragg diffraction.

3.2.1.1 How does an AOM work?

Sound is a dynamic strain involving molecular vibrations that take
the form of waves that travel at a velocity characteristic of the medium
i.e the velocity of the sound. Sound that travels through a crystal can
be modelled as crests of increased refractive index alternating with
troughs of decreased refractive index. Light incident on gradients
in refractive index is scattered, therefore the light scatters from the
acoustic wavefronts. In an AOM the light scattered from successive
wavefronts interferes constructively. Figure 8 shows two rays of light
impinging on two consecutive wavefronts in a crystal. Note that only
some of the light is scattered from these wavefronts[16]. The optical
and acoustic wavelengths are denoted by $\lambda_L$ and $\Lambda$ respectively, while
$\theta_i$ and $\theta_d$ are the angles the incident and scattered light rays make
with the acoustic wavefronts respectively. The condition for construc-
tive interference of the scattered light is

$$n\lambda_L = \Lambda(\sin\theta_i + \sin\theta_d),$$  \hspace{1cm} (10)

where n is an integer [17]. For an optical wave scattering from sound
with a frequency of order $10^8$ Hz, it can be shown that the conserva-
tion of energy and momentum requires that $\theta_i=\theta_d[18]$ , and so we
can approximate equation 1 as

$$n\lambda_L = 2\Lambda(\sin\theta_d).$$  \hspace{1cm} (11)

The Bragg condition would imply that there is only one value of the
deflection angel $\Theta=2\theta_d$, but this is based on the assumption that the
acoustic and optical wave fronts are plane waves (i.e, infinitely wide)
i.e, the source should be kept very far from the experiment. If the
acoustic beam is of finite width and the system is optimized for the
first order maximum beam power, i.e. for the Bragg condition with
$n=1$, the result will be some light with scattering angle $m\Theta$, where m
is the integer corresponding to the other orders[19].

The important element of an AOM is transparent crystal through
which light propagates. A piezoelectric transducer attached to the
crystal is used to excite a sound wave with a frequency of the order
of 100 MHz.

The intensity of the reflected light in a Bragg cell is proportional
to the intensity of sound, if the sound intensity is sufficiently weak.
Using an electrically controlled acoustic transducer (an AOM driver),
the intensity of the reflected light can be varied proportionally. The
device can be used as a linear analog modulator of light[9].
3.2 Working Principles of Important Blocks

3.2.1.2 AOM Control

Within an AOM the acoustic wave is provided by a radio frequency (RF) signal to the AOM, controlled by an AOM driver. This driver has three components, a Voltage Controlled Oscillator (VCO); a Voltage Variable Attenuator (VVA); and an amplifier. A schematic diagram of this setup is shown in figure 9.

The VCO provides a RF sine-wave output, the frequency of the RF output is determined by an applied control voltage, and varies approximately linearly with it. The VVA attenuates the output from the VCO, the amount of attenuation is controlled by varying the applied control voltage to the VVA. The amplifier amplifies the output of the VVA, such that the RF output is sufficient to drive the AOM. The response of the AOM varies with the frequency and amplitude of the input RF signal. It is good practice to ensure that the amplifier is separated from the VCO to guard against heating from the amplifier affecting the VCO frequency, whilst ensuring the cable connecting the

Figure 8: The Bragg construction for identifying ray directions with constructive interference[16].

Figure 9: Schematic diagram of an AOM driver.
amplifier to the AOM is kept as short as possible to minimize any loss of signal or creation of noise.

For this system, an AOM and an AOM driver from IntraAction Corp (AOM ME-80 series) have been used. The driver can take a modulation input of 0-1 V and it has a manual knob to control the attenuation of the laser output intensity. First step in this stabilization system was dumping so that we can bring the laser intensity into a desired level and making sure that we are not going to damage any other components present in the system.

3.2.2 Photodiode

Photodetectors can convert incident light to an electrical signal. In many photodetectors such as photoconductors and photodiodes this conversion is typically achieved by the creation of free electron hole pairs (EHPs) by the absorption of photons, that is, the creation of electrons in the conduction band (CB) and holes in the valence band (VB). Photodiode-type devices are small and have high speed and good sensitivity for use in various optoelectronics applications, the most important of which is in optical communications\[20]\.

The intensity measured by the photodiode is the most critical element in the system to make the pulse energy stable and to give a precise signal to the feedback loop. A biased large area Si photodiode with an area of 75.4 mm\(^2\) and rise time in the orders of ns (Thorlabs DET-100A) was used to measure the pulse signal. This photodiode can be used from 350-1100 nm wavelength region. This PD was chosen not to damage the device by focusing too much on it with high intensity laser pulses. Some optical attenuators were also used to bring the intensity level further down to make sure photodiode won’t get saturated. Figure 10 shows the output of the large area photodiode used in the experiment, which was measured by a fast oscilloscope\(^1\) with an impedance of 1\(\text{M}\Omega\). There is a pulse for every ms which is the repetition time of the femtosecond laser system.

3.2.3 Analog Low pass filter

An analog Sallen-Key low pass filter was designed and implemented to shape the output signal of the photodiode and considerably, reduce the fluctuations of the signal.

3.2.3.1 Filter Design

An active analog low pass filter was designed based on Sallen-Key topology. Sallen-Key topology is sometimes referred as voltage controlled voltage source, or VCVS. Sallen-Key circuit defines an architecture or a circuit topology that can be used to realize various second-order transfer functions. Cascaded sections of these second order cir-

\(^1\) The fast oscilloscope used: LeCroy waveRunner 6050A 500 MHz Oscilloscope Quad 5 GS/s
cuits can realize higher even order filters. A general Sallen-Key topology circuit diagram is shown in figure 11.

Larger Q factors\(^2\) are attainable by using a positive feedback amplifier. If the positive feedback is controlled-localized to the cut-off frequency of the filter-almost any Q can be realized, limited mainly by the physical constraints of the power supply and component tolerances. Figure 11 shows a unity gain amplifier used in this manner. In 1955, R.P. Sallen and E.L. Key described these filter circuits, and hence they are generally known as Sallen-Key filters. It's transfer function is given below.

\[
\frac{V_o}{V_i} = \frac{(2\pi f_c)^2}{s^2 + 2\zeta(2\pi f_c)s + (2\pi f_c)^2}
\]  \hspace{1cm} (12)

where \(f_c\) is the cut-off frequency of the filter and \(\zeta\) is the damping ratio of the filter. The cut-off frequency \(f_c\) is given by

\[
f_c = \frac{1}{2\pi\sqrt{R_1C_1R_2C_2}}
\]  \hspace{1cm} (13)

The operation can be qualitatively described:

- At low frequencies, where \(C_1\) and \(C_2\) appear as open circuits, the signal is simply buffered to the output.

- At high frequencies, where \(C_1\) and \(C_2\) appear as short circuits, the signal is shunted to ground at the amplifier’s input.

\(^2\text{Q factor or Quality factor}\) is a dimensionless parameter that describes the damping nature of an oscillator or a resonator and characterizes a resonator’s bandwidth relative to its center frequency.
amplifier amplifies this input to its output, and the signal does not appear at $V_o$.

- Near the cut-off frequency, where the impedance of $C_1$ and $C_2$ is on the same order as $R_1$ and $R_2$, positive feedback through $C_2$ provides Q enhancement of the signal.

There are many simplified models to enhance the Q factor and make the analysis simple. I have chosen the very simplified model of the filter by selecting $R_1=R_2$ and $C_1=C_2$ so that the Q-factor becomes independent of the cut-off frequency.

This filter was designed for many cut-off frequencies starting with the repetition rate of the laser i.e, 1000 Hz and 500 Hz, 300 Hz and finally 100 Hz. $R_1$, $R_2$, $C_1$ and $C_2$ were chosen according to the cut-off frequencies of the filter. Firstly, a second order filter was implemented with a quality factor of 0.5 and identical sections were cascaded to increase the order of the filter further for better results. This was done for all the above stated cut-off frequencies but here I presented the final filter used in the experiment which is a unity gain Sallen-Key low pass filter with 100 Hz cut-off frequency (cf. Appendix D). This is compared with the 300Hz Sallen-Key low pass filter, the advantages and the disadvantages are discussed briefly.

The components of the active analog filter circuits were chosen so that it can give the better filtering response. Low noise operational amplifiers(LMC6044 CMOS Quad Micropower Operational amplifiers from Texas instruments), NPO type ceramic capacitors with negative temperature coefficient were selected to compensate the positive temperature coefficient of metal film resistors, which can reduce the sensitivity of active filters to resistance and capacitance change.

In active filters, the role of the operational amplifier is to provide amplification and isolation. Its closed-loop bandwidth has to be higher than the cutoff frequency of the filter at least 100 times and slew rate
should be larger than the product of cut-off angular frequency and output voltage peak-peak, which can guarantee that there is no signal distortion in the filter. Input bias current should be in the pico Ampere (pA) level in order to decrease the input offset voltage, which is also advantageous in selecting the component parameters of resistors and capacitors[21].

3.2.4 PID Controller Design

PID controller is a three action most widely used controller in process control. Here "P" represents proportional control, 'I' represents integral control and 'D' represents derivative control action. The PID controllers are by far the most dominating form of feedback in use today, because of their simplicity, performance, robustness and availability of many effective yet simple tuning methods based on minimum plant model knowledge. It has tuning constants which brings the process value as close to the desired operating point(set point). Setting the parameters of PID is called tuning of the PID controller. In most conditions, the requirement is that the controller should act in such a manner that the process value is as close to the set point as possible. The control engineer uses the PID algorithms to achieve this [22]. A simple structure of control is the feedback control structure as shown in figure 12.

![Figure 12: The feedback control structure][22]

The proportional action provides control signal that is proportional to the error between the reference signal and the actual output. The integral action provides integral signal of the error, while the derivative action provide derivative signal of the error. The relation between the control $u(t)$ and error $e(t)$ can be expressed in the following form:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt}e(t)$$

(14)

Where $K_p$ is the proportional gain, $K_i$ is the integral gain and $K_d$ is the derivative gain. All these three parameter have to be tuned to
the desired controller output. The corresponding transfer function is given by

$$K(s) = K_p + \frac{K_i}{s} + K_d s$$  \hspace{1cm} (15)

The block diagram of a PID controller is shown in figure 13.

Figure 13: Block diagram of PID controller with all its terms.

Proportional term in the controller increases the steady state error (SSE) of the controller then integral action comes into play to reduce the offset (decreases the SSE). Derivative term is to decrease the settling time and increase the stability of the system but in general the derivative systems amplify the high frequency noise of the controller hence, one has to employ an additional low pass filter in order to get rid of the high frequency. By keeping this fact in the mind a simple active analog PI controller was designed and implemented using operational amplifiers (LMC6044 CMOS Quad Micropower Operational Amplifier from Texas instruments), ceramic capacitors, some fixed resistors and potentiometers were used as variable resistors for tuning (cf. Appendix D for the electronic circuit). The output of this controller was given to the AOM as feedback but, we know that the AOM takes the voltages between 0-1 V. This was done by tuning the controller parameters manually by trial and error method.

---

3 Finding out the best possible result by changing the different controller gains such as proportional and integral gains.
The 100 Hz Sallen-Key low pass filter was simulated with 1kHz sinusoidal input in NI-Multisim circuit design software. The simulated response of the filter \( f_c = 100\text{Hz} \) with 1kHz sinusoidal input is shown in figure 14.

Figure 14: Simulated response of the 4\(^{th}\) order Sallen-Key filter with 1kHz sinusoidal signal as input signal.

In figure 14 blue line is the input signal i.e, the 1kHz sinusoidal signal and the red line is the filtered output response of the 100Hz 4\(^{th}\) order low pass filter. The same 100Hz 4\(^{th}\) order filter was implemented with electronic components and tested with 1kHz sinusoidal signal and the response is shown in figure 15. The response (red line) is comparable to the simulated response of the filter shown in figure 14 though it is fluctuating a bit around its mean value.

The response of this 100Hz Sallen-Key low pass filter was measured for a long time and Fourier transform of the data was calculated to get more details of the filtered data and the Fourier transform is shown in figure 16. This shows that there are no clear features 1000Hz. This means the output response is almost behaving like a DC signal i.e, the filtered response is not fluctuating or with minimum amount of fluctuations (as per the demand which is less than 1\%).

To get a clear idea of the filter response, the rise time (or response time) of the filter was measured by fast oscilloscope and the output was plotted as shown in figure 17.
The rise time\textsuperscript{1} of this 100 Hz filter is around 10 ms which is equal to the bandwidth\textsuperscript{2} of the filter with 100 Hz cut-off frequency. The lines are drawn just to indicate the pulses and each line indicate a femtosecond pulse per every millisecond (i.e the repetition rate of the kHz laser) with certain amplitude.

The laser system produces a train of femtosecond pulses, these measured pulses after the photodiode were shaped and spread using an analog active low pass Sallen-Key filter with 300 Hz cut-off frequency. The cut-off was chosen in order to get rid off the high frequency noise and to make the pulse fluctuations as small as possible. This filter brings the voltage level of the signal to a certain desired level with minimum fluctuations possible.

Figure 18 shows the response of the 6\textsuperscript{th} order filter with $f_c = 300$ Hz, the response is flat and looking like having no fluctuations but when we zoom in the response we can observe some amount of fluctuations. The filter was implemented with three 2\textsuperscript{nd} order identical sections of Sallen-Key filter with $f_c = 300$ Hz and the total order of the filter has become 6. To make the filter structure as simple as possible, the cut off frequency has been further reduced to 100 Hz. The filter designed with 100 Hz was realized by using only two identical 2\textsuperscript{nd} order Sallen-Key filter sections (cf. Appendix D) with $f_c = 100$ Hz.

The 2\textsuperscript{nd} order Sallen-Key filter ($f_c = 100$Hz) output response was measured for a long time about 5s with 7 different traces. The average of all the traces was calculated and plotted as shown in figure 19. The same has done for the 4\textsuperscript{th} order filter as well and the output response is shown in figure 20. When we compare the output in the figure 20

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure15.png}
\caption{Measured response of the 4\textsuperscript{th} order Sallen-Key filter with 1kHz sinusoidal signal as the input signal.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure18.png}
\caption{Response of the 6\textsuperscript{th} order filter with $f_c = 300$ Hz.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure19.png}
\caption{Output response of the 2\textsuperscript{nd} order Sallen-Key filter ($f_c = 100$Hz).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure20.png}
\caption{Output response of the 4\textsuperscript{th} order Sallen-Key filter.}
\end{figure}

\textsuperscript{1} Rise time is the time taken by the signal to change from 5\% to 95\% of its final value
\textsuperscript{2} The bandwidth of a low pass filter is equal to the cut off frequency of the filter.
with the output in the figure 18, the signal level has been increased due to the order of the filter and the response is similar to the 6th order 300Hz low pass filter. This means the same result has been achieved with the simpler filter structure. Table 1 shows comparison between two types of filters with different parameters.

<table>
<thead>
<tr>
<th>Cutoff Frequency</th>
<th>Order</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 Hz</td>
<td>6</td>
<td>2.5868</td>
<td>0.17%</td>
</tr>
<tr>
<td>100 Hz</td>
<td>2</td>
<td>4.8528</td>
<td>0.17%</td>
</tr>
<tr>
<td>100 Hz</td>
<td>4</td>
<td>4.8668</td>
<td>0.16%</td>
</tr>
</tbody>
</table>

Table 1: Comparison between two filters with different parameters.

Frequency domain analysis was also performed on the time domain data measured for 100 Hz Sallen-Key low pass filter to get the clear picture of the frequency components present in the data measured. Firstly, the Fourier transform of the 2nd order 100 Hz low pass filter was calculated with the time domain data measured for 7 different traces and the average of these traces was plotted as shown in figure 21.

When we observe figure 21a, there is a peak at 1000 Hz frequency, which shows that the filter response is still oscillating with the repetition rate of the frequency. Also, there are peaks at very low frequencies, those peaks can be seen in figure 21b. The considerable peaks are present below or just above 100 Hz frequency which is the cut off frequency of the low pass filter. The Fourier transform of the 4th
order 100 Hz low pass filter was also calculated and plotted for 7 different traces of the time domain data shown in figure 22.

Now, the peak at 1000 Hz has disappeared as we observe figure 22a and the time domain response is no longer oscillating. The peaks at low frequencies are still present below or just above the 100 Hz frequency in figure 22b. Hence, one can guess these peaks are present due to the filtered laser pulses but can not be guaranteed because these can be coming from the electronics noise of the low pass filter. To make sure that these peaks are due to the filtered laser pulses. We can compare this with the Fourier transform of the filtered response of the low pass filter with 1kHz sinusoidal signal shown in figure 16.

Figure 17: Response time of the filter measured by fast oscilloscope at 2 shown in figure 7. Rise time (Response time) is measured by blocking the filtered signal and released by keeping the time scale of the oscilloscope in seconds.

Figure 18: Response of the 6th order sallen-key filter (f_c=300 Hz) measured by fast oscilloscope at 2 as shown in figure 7.
Figure 19: Response of the 2\textsuperscript{nd} order sallen-key filter (\(f_c=100\) Hz) measured by fast oscilloscope at 2 as shown in figure 7.

Figure 20: Response of the 4\textsuperscript{th} order sallen-key filter (\(f_c=100\) Hz) measured by fast oscilloscope at 2 as shown in figure 7.

Figure 21: Fourier transform of the 2\textsuperscript{nd} order sallen-key filter (\(f_c=100\) Hz).

Figure 22: Fourier transform of the 4\textsuperscript{th} order sallen-key filter (\(f_c=100\) Hz).
From figure 16, it can be seen that there are no clear features below 1000 Hz. Therefore, it is assumed that the peaks which are present in figure 22b originate from the filtered laser pulses.

The filtered output signal will be sent to an analog PI controller which was designed and the output of the PI controller will act as feedback signal or the modulation input signal to the AOM driver. As we discussed in previous chapter, the AOM would take the modulation input ranging between 0 to 1 V. This would be achieved by manually tuning the control parameters of the controller by trial and error method. Due to the time constraints the experiment was not continued further and will be continued in near future.

One more thing to note is that the output of the laser system changes day by day because of the requirements of energy for other projects, it is operated at different energies. Therefore, the stabilization system should be very flexible to adopt for these conditions everyday.
CONCLUSIONS

The main aim of this research project was to correct the power fluctuations of a Ti:Sapphire femtosecond laser present in Lund Laser Center in Lund University. A stabilization system was designed in order to achieve this. Though, the complete experiment was not performed as expected, the active attenuation and the active low pass filter are working up to the expectations and minimum fluctuations (<1%) were achieved. The filter design might be used in the future. This laser system is very sensitive to the noise such as mechanical noise due to the mechanical vibrations of cryo-cooling system, thermal noise due to temperature variations. The results presented in chapter 4 indicate that the 50Hz power line clearly affect the laser system. This fluctuations could be possible to mitigate with the presented design.

The designed analog PI controller might be tested in near future and there are many technical challenges to study. We might expect some technical problems with analog electronics design, e.g., electronic noise due to the electronic components. By keeping this in mind, designing the controller digitally is one of the other future ideas, instead of designing an analog PI controller. That can get rid of some of the noise from analog electronics. Implanting another filter after the controller is an idea to reduce the noise if any added by the electronics. Noise analysis is also a good idea to know the effect of the noise and then it’s possible to reduce by some technical measures. Otherwise, there are commercially available laser intensity controllers for example, New Focus LB1005 High-Speed Servo Controller. The LB1005 is a high-performance, standalone instrument that provides the appropriate analog signal conditioning for achieving stable feedback control. The LB1005 is particularly suited for high bandwidth filtering required to stabilize the intensity and frequency of many laser systems. The cost of this controller is very expensive for this type of applications as we compare this with designing in analog or digital electronics. The whole system can be developed with even lesser cost than the cost of the controller.

This stabilization system is a simple system to stabilize laser intensity. Most of the research groups in the world follow the same principle to stabilize the intensity of a laser i.e., the active attenuation, filtering and controlling with feedback. The design of the sections in the stabilization system would differ from group to group. The same experiment was performed in Lund laser center on the same laser system by a master student from Paris-Sud University in 2015 and achieved 5% of fluctuations and the noise also was affecting the whole system. Now, the amount of fluctuations were further reduced to <1% i.e., the requirement of the laser system but the final result is not what we have expected.


The atomic physics division is one of the research divisions in the department of physics, Lund University, Sweden. One of the leading research divisions in Europe when it comes to the field of laser matter interactions. Which uses most of the Lund high power laser facilities present in Lund Laser Center (LLC), founded in 1992, which is a European Major Research Infrastructure, this Facility is open not only to Swedish scientists, but also to scientists all over the world.

<table>
<thead>
<tr>
<th>Laser name</th>
<th>Repetition rate</th>
<th>Pulse energy</th>
<th>Pulse duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terawatt laser</td>
<td>10 Hz</td>
<td>1.5 J</td>
<td>40 fs</td>
</tr>
<tr>
<td>KHz laser</td>
<td>1 KHz</td>
<td>5 mJ</td>
<td>20 fs</td>
</tr>
<tr>
<td>OPCPA</td>
<td>200 KHz</td>
<td>10 mJ</td>
<td>7 fs</td>
</tr>
<tr>
<td>VUV</td>
<td>10 Hz</td>
<td>mJ</td>
<td>1 ns</td>
</tr>
</tbody>
</table>

Table 2: Lund High-power Laser facilities[Atomic physics website].

The first two lasers are based upon amplification in Ti:Sapphire. They have been constantly upgraded since their installation in 1992 and 1998. The third laser, which is just one year old, is based upon a different concept: OPCPA. It combines optical parametric amplification in a crystal and chirped pulse amplification (CPA) where an optical pulse is stretched in time, amplified and then compressed again. These lasers have the same nominal wavelength (800 nm) but vastly different pulse energy and repetition rate.
High harmonic generation (HHG) is a technique for producing spatially and temporally coherent extreme-ultraviolet (EUV) light, as well as light pulses as short as hundreds of attoseconds. To support pulses at such short duration light with a frequency higher than the inverse pulse duration is required corresponding to EUV wavelength of tenth of nanometers and shorter.

HHG occurs when an intense pulsed laser beam is focused into a (noble)gas jet or solid. The intensity of the laser light is chosen such that its electric field amplitude is comparable to the electric field in atoms. Such fields are able to detach electrons from atoms by tunnel ionization, as opposed to photo-ionization by a weak field with high enough photon energy. The detached electron is accelerated in the field and under certain conditions has significant probability to hit the ion left behind upon return. The "collision" results in the emission of high energy photons. This description is called three step model and depicted in Figure 23.

![Figure 23: Three step model of HHG. 1) Tunneling 2) Acceleration 3) Recombination.](image)

The high harmonic spectrum has characteristic features (Figure 24). That is, its frequency range extends well beyond the ionization potential $I_p$ of the source atom with fairly uniform intensity (the plateau) and then abruptly falls off (the cutoff). The spacing between the harmonics is regular and precisely twice as large as the driving laser frequency.
Figure 24: High Harmonic Generation Spectrum [23].
Ultrafast optics means not only short pulses but also high intensity. This was achieved by the concept of chirped pulse amplification (CPA), which includes stretching the pulse, amplification in multiple passes with different amplification stages in tandem and then compressing the pulse to as short as possible. The schematic representation of the chirped pulse amplification is shown in Figure 25.

**Grating Stretcher:** Which uses two grating components and pulses would go under group delay dispersion (GDD), which leads highly chirped and stretched pulses. GDD is second order chromatic dispersion, the second derivative of the spectral phase change. GDD can be either negative or positive. Actually, stretcher would make the pulse peak power lower than before by preserving the bandwidth according to the stretching factor, typically $10^3 - 10^4$.

**Amplification:** Stretched pulses would be amplified using single or multiple amplification stages with one or multiple number of passes. There would be no problems with non-linearities in the active medium of the amplifier because of the stretching and no damage to the crystal.

**Compression:** Once the pulses achieve the desired energy level, they are sent through a second GDD optical line having the opposite sign of the dispersion coefficient with respect to the stretcher. With proper matching of the two GDDs, the pulses can be compressed down to the bandwidth limit.

### C.1 Chirped Gaussian Pulse

A more general Gaussian pulse has a complex envelope $A(t) = A_0 \exp(-\alpha t^2)$, where $\alpha = (1 - ja)/\tau^2$ is a complex parameter and $\tau$ and $a$ are real parameters, so that

$$A(t) = A_0 \exp(-t^2/\tau^2) \exp(jat^2/\tau^2).$$

The magnitude of the complex envelope is a Gaussian function $|A_0|\exp(-t^2/\tau^2)$ and the intensity is also Gaussian. The phase is a quadratic function.
\[ \varphi = \frac{at^2}{\tau^2} \] so that the instantaneous frequency \( \nu_i = \nu_0 + \frac{at}{\pi \tau^2} \) is a linear function of time; i.e., the pulse is linearly chirped with chirp parameter \( a \). The pulse is up-chirped for positive \( a \), down-chirped for negative \( a \), and transform-limited (Unchirped) \( a=0 \).
The first operational amplifier stage acts as a buffer stage between the photodiode and the low pass filter. Two second order filters cascaded to form a fourth order filter. Operational amplifiers provide buffering and isolation between the two sections. This circuit was designed in Multisim circuit design software.

A simple Proportional-Integral controller was designed as in Figure 29 based on the mathematical principles of control theory using operational amplifiers. Set point was given as a constant voltage supplied by a DC regulated power supply (RPS) as one of the inputs to the controller and the other input was the output of the active low pass filter.
Mallikarjun Madagalam